Problem 14)

$\boldsymbol{M}(\boldsymbol{r},t)=M_o\hat{\boldsymbol{z}}\,\text{Sphere}(r/R)$ is the precise representation of the magnetization distribution, which, in spherical coordinates, is written $\boldsymbol{M}(\boldsymbol{r},t)=M_o\text{Sphere}(r/R)(\cos\theta\hat{\boldsymbol{r}}-\sin\theta\hat{\boldsymbol{\theta}})$.

a) $$\rho_{\text{bound}}^{(m)}(\boldsymbol{r},t)=-\boldsymbol{\nabla}\cdot\boldsymbol{M}(\boldsymbol{r},t)=-\frac{1}{r^2}\frac{\partial(r^2M_r)}{\partial r}-\frac{1}{r\sin\theta}\frac{\partial(\sin\theta M_\theta)}{\partial\theta}$$

$$=-\frac{M_o\cos\theta}{r^2}[2r\,\text{Sphere}(r/R)-r^2\delta(r-R)]+\frac{M_o\,\text{Sphere}(r/R)}{r\sin\theta}(2\sin\theta\cos\theta)$$

$$=M_o\delta(r-R)\cos\theta.$$

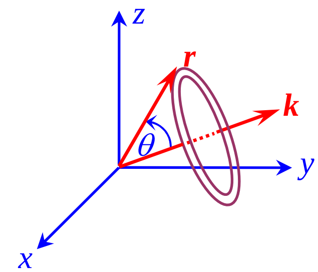
This surface-charge-density is positive on the upper hemisphere and negative on the lower hemisphere, changing continuously from maximum at the north-pole, to zero at the equator, to minimum at the south-pole.

b) $$\boldsymbol{J}_{\text{bound}}^{(e)}(\boldsymbol{r},t)=\mu_o^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r},t)=\frac{\mu_o^{-1}}{r}\left[\frac{\partial(r\,M_\theta)}{\partial r}-\frac{\partial M_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

$$=\frac{\mu_o^{-1}M_o}{r}\{-[\text{Sphere}(r/R)-r\,\delta(r-R)]\sin\theta+\text{Sphere}(r/R)\sin\theta\}\hat{\boldsymbol{\phi}}$$

$$=\mu_o^{-1}M_o\delta(r-R)\sin\theta\hat{\boldsymbol{\phi}}.$$

This azimuthal surface-current-density is zero at the north-pole, increases to a maximum at the equator, then decreases again to zero at the south-pole.



c) $$\boldsymbol{M}(\boldsymbol{k},\omega)=\int_{-\infty}^{\infty}M_o\hat{\boldsymbol{z}}\,\text{Sphere}(r/R)\exp[-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]\mathrm{d}\boldsymbol{r}\mathrm{d}t$$

$$=M_o\hat{\boldsymbol{z}}[2\pi\delta(\omega)]\int_{r=0}^{R}\int_{\theta=0}^{\pi}\exp(-\mathrm{i}kr\cos\theta)2\pi r^2\sin\theta\,\mathrm{d}r\mathrm{d}\theta$$

$$=4\pi^2M_o\hat{\boldsymbol{z}}\,\delta(\omega)\int_{r=0}^{R}\frac{\exp(-\mathrm{i}kr\cos\theta)}{\mathrm{i}kr}\bigg|_{\theta=0}^{\pi}r^2\mathrm{d}r$$

$$=8\pi^2M_ok^{-1}\delta(\omega)\hat{\boldsymbol{z}}\int_{r=0}^{R}r\sin(kr)\mathrm{d}r$$

Integration by parts →

$$=8\pi^2M_ok^{-1}\delta(\omega)\hat{\boldsymbol{z}}\left[-k^{-1}r\cos(kr)\Big|_{r=0}^{R}+\int_{r=0}^{R}k^{-1}\cos(kr)\mathrm{d}r\right]$$

$$=8\pi^2M_ok^{-1}\delta(\omega)\hat{\boldsymbol{z}}\left[-k^{-1}R\cos(kR)+k^{-2}\sin(kR)\right]$$

$$=8\pi^2M_ok^{-3}[\sin(kR)-kR\cos(kR)]\delta(\omega)\hat{\boldsymbol{z}}.$$