Problem 13)

a) $\vec{M}(\vec{r},t) = M_0 \,\mathrm{Circ}(r/R)\,\mathrm{Rect}(z/L)\cos(\omega_0 t)\,\hat{z}$

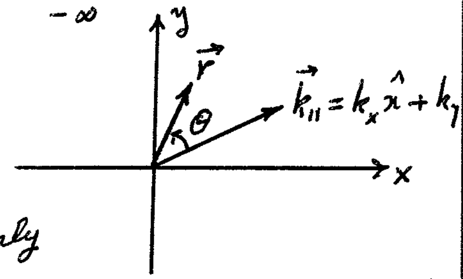
$$\vec{J}_b(\vec{r},t) = \frac{1}{\mu_0}\vec{\nabla}\times\vec{M}(\vec{r},t) = -\frac{1}{\mu_0}\frac{\partial M_z}{\partial r}\hat{\phi} = \frac{M_0}{\mu_0}\delta(r-R)\,\mathrm{Rect}(z/L)\cos(\omega_0 t)\,\hat{\phi}$$

Note: $\hat{\phi} = \hat{z}\times\hat{r}$

b) $$\vec{J}_b(\vec{k},\omega) = \int_{-\infty}^{\infty}\vec{J}_b(\vec{r},t)\,e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\,d\vec{r}\,dt$$

$$= \frac{M_0\hat{z}}{\mu_0}\times\int_{r=0}^{\infty}\int_{\theta=0}^{2\pi}\hat{r}\,\delta(r-R)\,e^{-ik_\parallel r\cos\theta}\,r\,d\theta\,dr\int_{-\infty}^{\infty}\mathrm{Rect}(z/L)\,e^{-ik_z z}\,dz$$

$$\int_{-\infty}^{\infty}\frac{e^{i\omega_0 t}+e^{-i\omega_0 t}}{2}\,e^{i\omega t}\,dt$$



Symmetry between θ and $-\theta$ means that the only contribution of $\hat{r}$ is its projection along $\vec{k}_\parallel$. Therefore, $\hat{r}$ can be replaced with $\cos\theta\,\hat{k}_\parallel$. We will have

$$\vec{J}_b(\vec{k},\omega) = \frac{M_0}{\mu_0}(\hat{z}\times\hat{k}_\parallel)\int_{r=0}^{\infty}r\,\delta(r-R)\left(\int_{\theta=0}^{2\pi}\cos\theta\, e^{-ik_\parallel r\cos\theta}\,d\theta\right)dr\left[\frac{e^{-ik_z L/2}-e^{+ik_z L/2}}{-ik_z}\right]$$

$$\times\frac{1}{2}\left[2\pi\,\delta(\omega+\omega_0) + 2\pi\,\delta(\omega-\omega_0)\right]$$

$$= \frac{\pi M_0}{\mu_0}(\hat{z}\times\hat{k}_\parallel)\left[-i2\pi\int_0^{\infty}r\,\delta(r-R)\,J_1(k_\parallel r)\,dr\right]\frac{2\sin(\frac{1}{2}Lk_z)}{k_z}\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$$

$$\vec{J}_b(\vec{k},\omega) = i\,\frac{4\pi^2 M_0 R}{\mu_0}(\hat{k}_\parallel\times\hat{z})\,J_1(k_\parallel R)\,\frac{\sin(Lk_z/2)}{k_z}\left[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right]$$