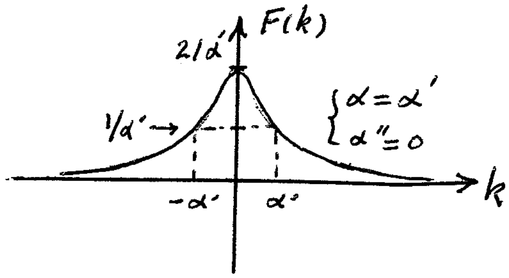
Problem 3)

$$\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-ikx} dx = \int_{-\infty}^{0} e^{\alpha x} e^{-ikx} dx + \int_{0}^{\infty} e^{-\alpha x} e^{-ikx} dx$$

$$= \int_{-\infty}^{0} e^{(\alpha - ik)x} dx + \int_{0}^{\infty} e^{-(\alpha+ik)x} dx = \frac{1}{\alpha - ik} + \frac{1}{\alpha + ik} = \frac{2\alpha}{\alpha^2 + k^2} \; ; \quad Re(\alpha) > 0$$

$$\mathcal{F}^{-1}\left\{\frac{2\alpha}{k^2+\alpha^2}\right\} = e^{-\alpha|x|} \Rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2\alpha}{k^2+\alpha^2} e^{ikx} dk = \frac{\alpha}{\pi}\int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2+\alpha^2} dk = e^{-\alpha|x|}$$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2+\alpha^2} dk = (\pi/\alpha) e^{-\alpha|x|}$$ ← Same as G.R. 3.389-5



$$\int_{-\infty}^{\infty} F(k) dk = \int_{-\infty}^{\infty} \frac{2\alpha'}{k^2+\alpha'^2} dk = 4\int_{0}^{\infty} \frac{dx}{x^2+1}$$

$x = k/\alpha'$

$$= 4\int_{0}^{\pi/2} \frac{1+\tan^2\theta}{1+\tan^2\theta} d\theta = 4\int_{0}^{\pi/2} d\theta = 2\pi$$

$x = \tan\theta$

When $\alpha' \to 0$, $F(k)$ becomes tall and narrow, centered around $k=0$ with an area equal to 2π. Therefore, $F(k) \to 2\pi\,\delta(k)$.