Problem 2)

a) $$\hat{f}(t) = \frac{1}{2\pi}\int_{-\Omega}^{\Omega}\left\{\int_{t_1}^{t_2} f(t')e^{-i\omega t'}dt'\right\}e^{+i\omega t}d\omega =$$

$$\frac{1}{2\pi}\int_{t_1}^{t_2} f(t')\left\{\int_{-\Omega}^{\Omega} e^{i\omega(t-t')}d\omega\right\}dt' = \frac{1}{2\pi}\int_{t_1}^{t_2} f(t')\left.\frac{e^{i\omega(t-t')}}{i(t-t')}\right|_{-\Omega}^{\Omega}dt'$$

$$= \int_{t_1}^{t_2} f(t')\frac{e^{i\Omega(t-t')}-e^{-i\Omega(t-t')}}{2\pi i(t-t')}dt' = \int_{t_1}^{t_2} f(t')\frac{\sin[\Omega(t-t')]}{\pi(t-t')}dt'$$

$$= \int_{t_1}^{t_2} f(t')\left(\frac{\Omega}{\pi}\right)\frac{\sin[\Omega(t-t')]}{\Omega(t-t')}dt' \Rightarrow \hat{f}(t) = \int_{t_1}^{t_2} f(t')\left(\frac{\Omega}{\pi}\right)\text{Sinc}\left[\frac{\Omega}{\pi}(t-t')\right]dt'$$

b) Sinc(t) is peaked at $t=0$, is symmetric, and has unit area. Therefore, $\left(\frac{\Omega}{\pi}\right)\text{Sinc}\left(\frac{\Omega}{\pi}t\right)$ is tall, narrow, and has unit area, which means that, in the limit $\Omega\to\infty$, the function $\frac{\Omega}{\pi}\text{Sinc}\left(\frac{\Omega}{\pi}t\right)\to\delta(t)$. We thus have, for sufficiently large Ω,

$$\hat{f}(t) \simeq \int_{t_1}^{t_2} f(t')\delta(t-t')dt' \xrightarrow[\text{Property of }\delta(\cdot)]{\text{using the sifting}} \hat{f}(t)\xrightarrow[\Omega\to\infty]{} f(t);\quad t_1<t<t_2$$

c) When $t<t_1$ or $t>t_2$, the function $\left(\frac{\Omega}{\pi}\right)\text{Sinc}\left[\frac{\Omega}{\pi}(t-t')\right]$ is peaked at $t'=t$, which is outside the interval (t_1,t_2). Within the (t_1,t_2) interval, therefore, the sinc function is zero yielding $\hat{f}(t)=0$.