Problem 2.67) The $\boldsymbol{E}$ and $\boldsymbol{H}$ fields of the first plane-wave are written

$$\boldsymbol{E}_1(\boldsymbol{r},t) = E_x\hat{\boldsymbol{x}}\cos[(\omega/c)(\sin\theta\, y + \cos\theta\, z - ct) + \varphi_1], \tag{1a}$$

$$\boldsymbol{H}_1(\boldsymbol{r},t) = Z_0^{-1}E_x(\cos\theta\,\hat{\boldsymbol{y}} - \sin\theta\,\hat{\boldsymbol{z}})\cos[(\omega/c)(\sin\theta\, y + \cos\theta\, z - ct) + \varphi_1]. \tag{1b}$$

Similarly, the $\boldsymbol{E}$ and $\boldsymbol{H}$ fields of the second plane-wave are given by

$$\boldsymbol{E}_2(\boldsymbol{r},t) = -E_x(\cos\theta\,\hat{\boldsymbol{y}} + \sin\theta\,\hat{\boldsymbol{z}})\cos[(\omega/c)(-\sin\theta\, y + \cos\theta\, z - ct) + \varphi_2], \tag{2a}$$

$$\boldsymbol{H}_2(\boldsymbol{r},t) = Z_0^{-1}E_x\hat{\boldsymbol{x}}\cos[(\omega/c)(-\sin\theta\, y + \cos\theta\, z - ct) + \varphi_2]. \tag{2b}$$

The Poynting vector of the superposition may thus be written straightforwardly, as follows:

$$\begin{aligned}\boldsymbol{S}(\boldsymbol{r},t) &= (\boldsymbol{E}_1 + \boldsymbol{E}_2)\times(\boldsymbol{H}_1 + \boldsymbol{H}_2)\\ &= Z_0^{-1}E_x^2\{(\sin\theta\,\hat{\boldsymbol{y}} + \cos\theta\,\hat{\boldsymbol{z}})\cos^2[(\omega/c)(\sin\theta\, y + \cos\theta\, z - ct) + \varphi_1]\\ &\qquad -(\sin\theta\,\hat{\boldsymbol{y}} - \cos\theta\,\hat{\boldsymbol{z}})\cos^2[(\omega/c)(-\sin\theta\, y + \cos\theta\, z - ct) + \varphi_2]\\ &\qquad +2\sin\theta\cos\theta\,\hat{\boldsymbol{x}}\cos[(\omega/c)(-\sin\theta\, y + \cos\theta\, z - ct) + \varphi_2]\\ &\qquad\qquad \times\cos[(\omega/c)(\sin\theta\, y + \cos\theta\, z - ct) + \varphi_1]\}.\end{aligned} \tag{3}$$

The above expression for $\boldsymbol{S}(\boldsymbol{r},t)$, when simplified and rearranged, yields

$$\begin{aligned}\boldsymbol{S}(\boldsymbol{r},t) = Z_0^{-1}E_x^2\cos\theta\,\{&\hat{\boldsymbol{z}} + \sin\theta\cos[2(\omega/c)\sin\theta\, y + (\varphi_1 - \varphi_2)]\hat{\boldsymbol{x}}\\ &+\sin\theta\cos[2(\omega/c)\cos\theta\, z - 2\omega t + (\varphi_1 + \varphi_2)]\,\hat{\boldsymbol{x}}\\ &-\tan\theta\sin[2(\omega/c)\sin\theta\, y + \tfrac{1}{2}(\varphi_1 - \varphi_2)]\\ &\qquad\times\sin[2(\omega/c)\cos\theta\, z - 2\omega t + \tfrac{1}{2}(\varphi_1 + \varphi_2)]\,\hat{\boldsymbol{y}}\\ &+\cos[2(\omega/c)\sin\theta\, y + \tfrac{1}{2}(\varphi_1 - \varphi_2)]\\ &\qquad\times\cos[2(\omega/c)\cos\theta\, z - 2\omega t + \tfrac{1}{2}(\varphi_1 + \varphi_2)]\,\hat{\boldsymbol{z}}\}.\end{aligned} \tag{4}$$

Finally, upon time-averaging, we find

$$\langle\boldsymbol{S}(\boldsymbol{r},t)\rangle = Z_0^{-1}E_x^2\cos\theta\,\{\hat{\boldsymbol{z}} + \sin\theta\cos[2(\omega/c)\sin\theta\, y + (\varphi_1 - \varphi_2)]\,\hat{\boldsymbol{x}}\}. \tag{5}$$

The time-averaged component of the Poynting-vector along $\hat{\boldsymbol{x}}$ is thus seen to oscillate along the y-axis and have, relative to its z-component, a magnitude proportional to $\sin\theta$. The presence of a net Poynting vector along $\hat{\boldsymbol{x}}$ is rather surprising, considering that the two beams do not form interference fringes, as their polarization directions are orthogonal to each other.