Problem 2.63) a) The surface-current-density $\boldsymbol{J}_s$ is the product of ordinary current-density $\boldsymbol{J}_{\text{free}}$ and the *very small* thickness τ of the cylindrical shell, that is, $\boldsymbol{J}_s = \boldsymbol{J}_{\text{free}}\tau$. Considering that the units of $\boldsymbol{J}_{\text{free}}$ are ampere/m^2, we conclude that $\boldsymbol{J}_s$ has units of ampere/m.

b) In a cylindrical coordinate system the H-field has three components, namely H_ρ, H_φ, and H_z. Due to the symmetry of the setup, these field components must be independent of the φ and z coordinates. This is because the current-carrying cylinder would look exactly the same if the ρ and z coordinates of the observation point were fixed while its φ coordinate varied. Similarly, the system would look the same if the ρ and φ coordinates of the observation point were fixed while its z coordinate varied. The H-field components, therefore, can only be functions of the ρ coordinate.

c) In conjunction with Maxwell's 4th equation, $\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0$, we use a cylindrical volume of radius ρ and length L, centered on the z-axis, to show that $H_\rho(\rho) = 0$. Clearly, $H_\varphi(\rho)$ does not contribute to the surface integral of $\boldsymbol{B} = \mu_0\boldsymbol{H}$ over the cylinder. Also, contributions from $H_z(\rho)$ to the top and bottom facets of the cylinder cancel each other out. The contribution of $H_\rho(\rho)$ to the surface integral is $2\pi\rho L H_\rho(\rho)$, but the overall surface integral must be zero and, therefore, $H_\rho(\rho) = 0$.

Next, we use a circular loop of radius ρ in the xy-plane in conjunction with Maxwell's 2nd equation, $\boldsymbol{\nabla}\times\boldsymbol{H} = \boldsymbol{J}_{\text{free}}$, to demonstrate that $H_\varphi(\rho) = 0$. The line integral of H_φ around the loop is $2\pi\rho H_\varphi(\rho)$. However, no current crosses the loop and, therefore, $H_\varphi(\rho) = 0$.

Finally, we use rectangular loops in the ρz-plane, again in conjunction with Maxwell's 2nd equation, to obtain information about $H_z(\rho)$. If the rectangular loop is placed entirely outside the cylinder we find that no current crosses the loop and that, therefore, H_z outside the cylinder is uniform. Similarly, placing the rectangular loop inside the cylinder shows that H_z inside the cylinder is uniform as well. However, if the $L\times W$ rectangle has one leg inside and the opposite leg outside the cylinder, the integral of H_z around the loop will be $[H_z^{(\text{inside})} - H_z^{(\text{outside})}]L$, while the current that crosses the loop will be $J_{s0}L$. Consequently, $H_z^{(\text{inside})} - H_z^{(\text{outside})} = J_{s0}$.

Given that a uniform H_z field residing in the entire space cannot, in any way, be related via Maxwell's equations to the solenoidal current $J_{s0}\hat{\boldsymbol{\varphi}}$, we conclude that $H_z^{(\text{outside})} = 0$. Therefore,

$$\boldsymbol{H}(\rho,\varphi,z) = \begin{cases} J_{s0}\hat{\boldsymbol{z}}; & 0 \le \rho < R, \\ 0; & \rho > R. \end{cases}$$