**Problem 2.62**) a) Using Gauss' law in conjunction with spherical surfaces of radius $\rho$, we find, since the $E$-field inside the metallic shell of the inner sphere must vanish, that the total charge on the interior surface of the inner sphere must be zero. Therefore, $\sigma_{12}=0$. The charge content of the inner sphere is thus distributed entirely on its outer surface, namely, $\sigma_{11}=\sigma_1$.

As for the outer sphere, we place the Gaussian surface inside its metallic shell. Since the $E$-field inside the metal must be zero, the total charge inside the Gaussian sphere must vanish, that is, $4\pi R_1^2\sigma_1+4\pi R_2^2\sigma_{22}=0$. This yields $\sigma_{22}=-(R_1/R_2)^2\sigma_1$. The remaining charge will then appear on the outer surface of the outer sphere, that is, $\sigma_{21}=\sigma_2-\sigma_{22}$.

b) Inside the small sphere the $E$-field is zero. The total charge on the inner shell is $Q=4\pi R_1^2\sigma_1$. Therefore, in the region between the two spheres,

$$\boldsymbol{E}_1(\rho)=(Q/4\pi\varepsilon_o\rho^2)\hat{\boldsymbol{\rho}}=(\sigma_1/\varepsilon_o)\,(R_1/\rho)^2\hat{\boldsymbol{\rho}};\qquad R_1<\rho<R_2.$$

Outside the large sphere, the fields of the two spheres are superimposed, that is,

$$\boldsymbol{E}_2(\rho)=[(\sigma_1/\varepsilon_o)\,(R_1/\rho)^2+(\sigma_2/\varepsilon_o)\,(R_2/\rho)^2]\hat{\boldsymbol{\rho}};\qquad \rho>R_2.$$

c) The potential difference (i.e., voltage) between the spheres is given by the integral of $\boldsymbol{E}_1(\rho)$ along the radial direction from $R_1$ to $R_2$, that is,

$$V_{12}=\int_{R_1}^{R_2}E_{1\rho}(\rho)\mathrm{d}\rho=\int_{R_1}^{R_2}(\sigma_1/\varepsilon_o)\,(R_1/\rho)^2\mathrm{d}\rho=-(\sigma_1R_1^2/\varepsilon_o)\,[(1/R_2)-(1/R_1)]$$

$$=(\sigma_1/\varepsilon_o)(R_1/R_2)(R_2-R_1).$$

Therefore,

$$C=Q/V=\frac{4\pi R_1^2\sigma_1}{(\sigma_1/\varepsilon_o)(R_1/R_2)(R_2-R_1)}=4\pi\varepsilon_oR_1R_2/(R_2-R_1).$$