

Problem 2.55) Consider a cylindrical can of arbitrary radius R and arbitrary length L centered on the wire, as shown in the figure below. We apply the integral form of Maxwell's 4th equation, $\nabla \cdot \mathbf{B} = 0$, to this can. Using Gauss's theorem, the integral form is found to be: $\oint \mathbf{B} \cdot d\mathbf{s} = 0$. In the absence of magnetism, we have $\mathbf{M} = 0$ and $\mathbf{B} = \mu_0 \mathbf{H}$. Consequently, Maxwell's 4th equation demands that $\oint \mathbf{H} \cdot d\mathbf{s} = 0$.

On the closed surface of the cylindrical can, only two components of the H -field contribute to the surface integral: (i) on the top and bottom facets, H_z has nonzero integrals; (ii) on the cylindrical surface, H_ρ makes a nonzero contribution to the integral. However, symmetry indicates that the contribution of H_z to the top facet is exactly cancelled out by its contribution to the bottom facet—because the value of H_z (whatever it may be) cannot depend on the z -coordinate. As for H_ρ , its magnitude must be the same everywhere on the cylindrical surface, again because of symmetry; its contribution to the surface integral will thus be $2\pi RLH_\rho$. The total integral of the H -field over the surface of the can is, therefore, $2\pi RLH_\rho$, which must be zero in accordance with Maxwell's 4th equation. We conclude that $H_\rho(\rho, \phi, z)$ must be zero everywhere.

