
Problem 2.53 a) The linear velocity of the spherical surface is $\mathbf{V}(\rho = R, \theta, \phi) = (R \sin \theta) \omega \hat{\boldsymbol{\phi}}$. Therefore, the surface current density is $\mathbf{J}_s(R, \theta, \phi, t) = \sigma_s \mathbf{V}(R, \theta, \phi) = (R \omega \sigma_s \sin \theta) \hat{\boldsymbol{\phi}}$. The units of \mathbf{J}_s are the units of R [m] times the units of ω [sec^{-1}] times the units of σ_s [coulomb/m²], namely, [ampere/m].

b) In spherical coordinates, the divergence of the vector field \mathbf{J}_s whose only component is along the ϕ -axis, is given by $\nabla \cdot \mathbf{J}_s = \frac{1}{R \sin \theta} \frac{\partial J_{s\phi}}{\partial \phi} = 0$. Since the surface-charge-density σ_s has no time-dependence, its derivative with respect to time is zero, that is, $\frac{\partial \sigma_s}{\partial t} = 0$. Clearly, $\nabla \cdot \mathbf{J}_s + \frac{\partial \sigma_s}{\partial t} = 0$.
