Problem 2.50) a) The amount of charge flowing per unit time through each and every cross-section perpendicular to the z-axis is given by

$$I_o = \lambda_o V. \tag{1}$$

Note that the units on both sides of the above equation are [coulomb/meter].

b) Application of Maxwell's 1st equation to a cylinder of radius r and unit height along z yields the E-field as $\boldsymbol{E}(r,\phi,z) = \lambda_o \hat{\boldsymbol{r}}/(2\pi\varepsilon_o r)$. Similarly, application of Maxwell's 2nd equation to a circular loop of radius r parallel to the xy-plane yields the H-field as $\boldsymbol{H}(r,\phi,z) = \lambda_o V \hat{\boldsymbol{\phi}}/(2\pi r)$.

c) $$\boldsymbol{\nabla}\cdot\boldsymbol{D} = \varepsilon_o \boldsymbol{\nabla}\cdot\boldsymbol{E} = \varepsilon_o \frac{\partial(rE_r)}{r\partial r} = \varepsilon_o \frac{\partial[r\lambda_o/(2\pi\varepsilon_o r)]}{r\partial r} = \frac{\partial(\lambda_o)}{2\pi r\partial r} = 0. \tag{2}$$

$$\boldsymbol{\nabla}\times\boldsymbol{H} = -\frac{\partial H_\phi}{\partial z}\hat{\boldsymbol{r}} + \frac{\partial(rH_\phi)}{r\partial r}\hat{\boldsymbol{z}} = \frac{\partial[r\lambda_o V/(2\pi r)]}{r\partial r}\hat{\boldsymbol{z}} = \frac{\partial(\lambda_o V)}{2\pi r\partial r}\hat{\boldsymbol{z}} = 0. \tag{3}$$

Considering that $\boldsymbol{J}_{\text{free}} = 0$ in the surrounding space, and that $\boldsymbol{D} = \varepsilon_o \boldsymbol{E}$ is time-independent, the right-hand side of the above equation is equal to $\boldsymbol{J}_{\text{free}} + \partial\boldsymbol{D}/\partial t$. Maxwell's 2nd equation is thus satisfied.

$$\boldsymbol{\nabla}\times\boldsymbol{E} = \frac{\partial E_r}{\partial z}\hat{\boldsymbol{\phi}} - \frac{\partial E_r}{r\partial\phi}\hat{\boldsymbol{z}} = \frac{\partial[\lambda_o/(2\pi\varepsilon_o r)]}{\partial z}\hat{\boldsymbol{\phi}} - \frac{\partial[\lambda_o/(2\pi\varepsilon_o r)]}{r\partial\phi}\hat{\boldsymbol{z}} = 0. \tag{4}$$

Since $\boldsymbol{B} = \mu_o \boldsymbol{H}$ is time-independent, the right-hand side of the above equation is equal to $-\partial\boldsymbol{B}/\partial t$ and, therefore, Maxwell's 3rd equation is satisfied.

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = \mu_o \boldsymbol{\nabla}\cdot\boldsymbol{H} = \mu_o \frac{\partial H_\phi}{r\partial\phi} = \mu_o \frac{\partial[\lambda_o V/(2\pi r)]}{r\partial\phi} = 0. \tag{5}$$

d) $$\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t)\times\boldsymbol{H}(\boldsymbol{r},t) = \frac{\lambda_o\hat{\boldsymbol{r}}}{2\pi\varepsilon_o r}\times\frac{\lambda_o V\hat{\boldsymbol{\phi}}}{2\pi r} = \frac{\lambda_o^2 V\hat{\boldsymbol{z}}}{4\pi^2\varepsilon_o r^2}. \tag{6}$$

Electromagnetic energy thus flows along the z-axis within the free space region surrounding the rod. The rate of flow of this energy drops with the square of radial distance from the rod.