Problem 2.49) a) At the mirror surface, we have $z = 0$ and the tangential E-field is along the x-axis. Adding the x-components of the incident and reflected E-fields, we find

$$E_x^{(\text{inc})} + E_x^{(\text{ref})} = E_\text{o} \cos\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\} - E_\text{o} \cos\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\} = 0.$$

Since the fields inside the perfectly-conducting mirror are zero, the continuity of the tangential E-field requires $E_x^{(\text{total})}$ at the front facet of the mirror to vanish. This is indeed the case for the tangential component of the E-field at $z = 0$.

b) At the front facet, we have $z = 0$ and the tangential H-field is along the y-axis. Adding the y-components of the incident and reflected H-fields, we find

$$H_y^{(\text{inc})} + H_y^{(\text{ref})} = 2(E_\text{o}/Z_\text{o}) \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\}.$$

Since the H-field within the perfectly-conducting mirror is zero, the discontinuity of H_y must be accounted for by the presence of a surface-current-density whose magnitude is equal to H_y at the mirror surface, and whose direction, while perpendicular to the H-field, follows the right-hand rule. We will have

$$\boldsymbol{J}_s(x, y, z = 0, t) = 2(E_\text{o}/Z_\text{o})\hat{\boldsymbol{x}} \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\}.$$

c) At the front facet, we have $z = 0$ and the perpendicular E-field is along the z-axis. Adding the z-components of the incident and reflected E-fields, we find

$$E_z^{(\text{inc})} + E_z^{(\text{ref})} = -2E_\text{o} \sin\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\}.$$

Since the E-field within the perfectly-conducting mirror is zero, the discontinuity of E_z must be accounted for by the presence of a surface-charge-density whose magnitude is equal to $\varepsilon_0 E_z$ at the mirror surface. We find

$$\sigma_s(x, y, z = 0, t) = 2\varepsilon_\text{o} E_\text{o} \sin\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\}.$$

d) Charge-current continuity equation:

$$\begin{aligned}\boldsymbol{\nabla}\cdot\boldsymbol{J}_s + \partial\sigma_s/\partial t = \partial J_{sx}/\partial x + \partial\sigma_s/\partial t &= 2\text{i}(\omega/c)\sin\theta(E_\text{o}/Z_\text{o}) \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\} \\ &\quad - 2\text{i}\omega\varepsilon_\text{o} E_\text{o} \sin\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\} \\ &= 2\text{i}\omega(\varepsilon_\text{o} - \varepsilon_\text{o}) E_\text{o} \sin\theta \exp\{\text{i}(\omega/c)[(\sin\theta)x - ct]\} = 0.\end{aligned}$$