Problem 2.48)

a) $\boldsymbol{\nabla}\cdot\boldsymbol{D}=\rho_{\text{free}} \;\rightarrow\; \varepsilon_o\boldsymbol{\nabla}\cdot\boldsymbol{E}=0 \;\rightarrow\; \partial E_z/\partial z=\partial[E_o\cos(k_o y-\omega_o t)]/\partial z=0.$ (1)

$\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}+\partial\boldsymbol{D}/\partial t \;\rightarrow\; -\partial H_x/\partial y=\varepsilon_o\partial E_z/\partial t$

$\rightarrow\; H_o k_o\sin(k_o y-\omega_o t)=\varepsilon_o E_o\omega_o\sin(k_o y-\omega_o t) \;\rightarrow\; H_o k_o=\varepsilon_o E_o\omega_o.$ (2)

$\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t \;\rightarrow\; \partial E_z/\partial y=-\mu_o\partial H_x/\partial t$

$\rightarrow\; -E_o k_o\sin(k_o y-\omega_o t)=-\mu_o H_o\omega_o\sin(k_o y-\omega_o t) \;\rightarrow\; E_o k_o=\mu_o H_o\omega_o.$ (3)

$\boldsymbol{\nabla}\cdot\boldsymbol{B}=0 \;\rightarrow\; \mu_o\boldsymbol{\nabla}\cdot\boldsymbol{H}=0 \;\rightarrow\; \partial H_x/\partial x=\partial[H_o\cos(k_o y-\omega_o t)]/\partial x=0.$ (4)

It is seen that Maxwell's 1^{st} and 4^{th} equations are already satisfied. As for the 2^{nd} and 3^{rd} equations, we note that Eq.(2) above yields $E_o/H_o=k_o/(\varepsilon_o\omega_o)$, whereas Eq.(3) yields $E_o/H_o=\mu_o\omega_o/k_o$. Consequently, we must have $k_o/(\varepsilon_o\omega_o)=\mu_o\omega_o/k_o$, which yields $k_o=\omega_o/c$. Substitution into either Eq.(2) or Eq.(3) now reveals that $E_o/H_o=Z_o$.

b) The discontinuity of $D_\perp=\varepsilon_o E_z$ at each surface is equal to the surface charge-density at that surface, that is,

$$\sigma_s(x,y,z=\pm\tfrac{1}{2}d,t)=\mp\,\varepsilon_o E_o\cos(k_o y-\omega_o t). \quad (5)$$

Similarly, the discontinuity of $H_{||}=H_x$ at each surface is equal to the surface current-density at the corresponding surface, with the current's direction being perpendicular to that of the H-field. We thus have

$$\boldsymbol{J}_s(x,y,z=\pm\tfrac{1}{2}d,t)=\mp\,H_o\cos(k_o y-\omega_o t)\hat{\boldsymbol{y}}. \quad (6)$$

c) At each surface, the charge-current continuity equation $\boldsymbol{\nabla}\cdot\boldsymbol{J}+\partial\rho/\partial t=0$ reduces to $\partial J_{sy}/\partial y+\partial\sigma_s/\partial t=0$. With the help of Eqs.(5) and (6), we write the continuity equation as follows:

$$\begin{aligned}\partial J_{sy}/\partial y+\partial\sigma_s/\partial t&=\pm\,H_o k_o\sin(k_o y-\omega_o t)\mp\varepsilon_o E_o\omega_o\sin(k_o y-\omega_o t)\\&=\pm\,(H_o k_o-\varepsilon_o E_o\omega_o)\sin(k_o y-\omega_o t)=0.\end{aligned} \quad (7)$$

In the last line of the above equation, we have used Eq.(2) to set $H_o k_o$ equal to $\varepsilon_o E_o\omega_o$.