Problem 2.44)

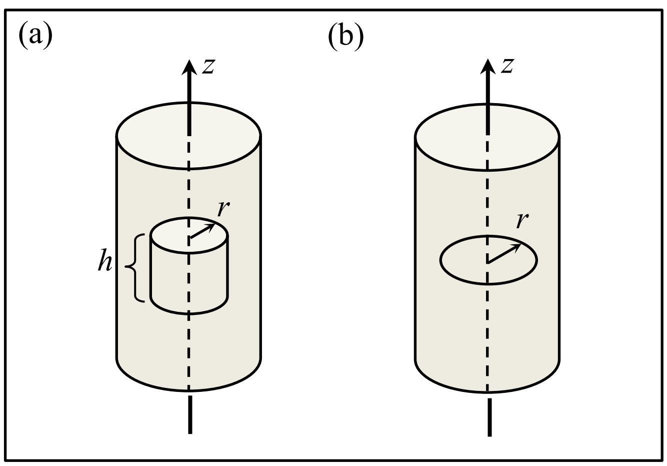
a) Symmetry dictates that the E-field be radial and independent of the azimuthal and vertical coordinates φ and z, that is, $\boldsymbol{E}(r,\varphi,z)=E_r(r)\hat{\boldsymbol{r}}$. Now, imagine a cylindrical surface of radius r and height h centered on the z-axis, as shown in figure (a) below. Application to this cylinder of the integral form of Maxwell's first equation $\oint_{\text{surface}}\varepsilon_0\boldsymbol{E}\cdot d\boldsymbol{s}=\int_{\text{volume}}\rho_{\text{free}}dv$ yields

$$2\pi rh\varepsilon_0 E_r(r)=\begin{cases}\pi r^2h\rho_0; & r\le R\\ \pi R^2h\rho_0; & r\ge R\end{cases}\quad\rightarrow\quad E_r(r)=\begin{cases}\rho_0 r/(2\varepsilon_0); & r\le R,\\ \rho_0R^2/(2\varepsilon_0 r); & r\ge R.\end{cases}$$

b) The current-density is the product of charge-density and velocity, that is, $\boldsymbol{J}=\rho_0 v_0\hat{\boldsymbol{z}}$.

c) Symmetry dictates that the H-field be azimuthal and independent of the azimuthal and vertical coordinates φ and z, that is, $\boldsymbol{H}(r,\varphi,z)=H_\varphi(r)\hat{\boldsymbol{\varphi}}$. Now, imagine a circular loop of radius r centered on the z-axis, as shown in figure (b) below. Application to this loop of the integral form of Maxwell's second equation $\oint_{\text{loop}}\boldsymbol{H}\cdot d\boldsymbol{\ell}=\int_{\text{surface}}\boldsymbol{J}\cdot d\boldsymbol{s}$ yields

$$2\pi rH_\varphi(r)=\begin{cases}\pi r^2\rho_0 v_0; & r\le R\\ \pi R^2\rho_0 v_0; & r\ge R\end{cases}\quad\rightarrow\quad H_\varphi(r)=\begin{cases}½\rho_0 v_0 r; & r\le R,\\ ½\rho_0 v_0 R^2/r; & r\ge R.\end{cases}$$



d) The Lorentz force-density is given by

$$\begin{aligned}\boldsymbol{f}(\boldsymbol{r},t)&=\rho(\boldsymbol{r},t)[\boldsymbol{E}(\boldsymbol{r},t)+\boldsymbol{v}(\boldsymbol{r},t)\times\boldsymbol{B}(\boldsymbol{r},t)]=\rho_0\big(E_r\hat{\boldsymbol{r}}+\mu_0 v_0\hat{\boldsymbol{z}}\times H_\varphi\hat{\boldsymbol{\varphi}}\big)\\ &=\rho_0\big(E_r\hat{\boldsymbol{r}}-\mu_0 v_0 H_\varphi\hat{\boldsymbol{r}}\big)=\rho_0[\rho_0 r/(2\varepsilon_0)-(\mu_0 v_0)(½\rho_0 v_0 r)]\hat{\boldsymbol{r}}\\ &=½\mu_0(c^2-v_0^2)\rho_0^2\, r\hat{\boldsymbol{r}}.\end{aligned}$$

It is readily observed that the radially outward push of the electric force is proportional to $c^2=(\mu_0\varepsilon_0)^{-1}$, whereas the radially inward pull of the magnetic force is proportional to v_0^2. These two forces will cancel out only if $v_0=c$.