Problem 2.43)

a) 1) $\boldsymbol{\nabla}\cdot\boldsymbol{D}=\rho_{\text{free}}$,

2) $\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}+\dfrac{\partial\boldsymbol{D}}{\partial t}$,

3) $\boldsymbol{\nabla}\times\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t}\quad\rightarrow\quad\boldsymbol{\nabla}\times(\varepsilon_{\text{o}}\boldsymbol{E}+\boldsymbol{P})=-\varepsilon_{\text{o}}\dfrac{\partial(\mu_{\text{o}}\boldsymbol{H}+\boldsymbol{M})}{\partial t}+\boldsymbol{\nabla}\times\boldsymbol{P}$

$\rightarrow\quad\boldsymbol{\nabla}\times\boldsymbol{D}=-\varepsilon_{\text{o}}\left(\dfrac{\partial\boldsymbol{M}}{\partial t}-\varepsilon_{\text{o}}^{-1}\boldsymbol{\nabla}\times\boldsymbol{P}\right)-\varepsilon_{\text{o}}\mu_{\text{o}}\dfrac{\partial\boldsymbol{H}}{\partial t}$

$\rightarrow\quad\boldsymbol{\nabla}\times\boldsymbol{D}=-\varepsilon_{\text{o}}\boldsymbol{J}_{\text{bound}}^{(\text{m})}-\varepsilon_{\text{o}}\mu_{\text{o}}\dfrac{\partial\boldsymbol{H}}{\partial t}$,

4) $\boldsymbol{\nabla}\cdot\boldsymbol{B}=0\quad\rightarrow\quad\mu_{\text{o}}\boldsymbol{\nabla}\cdot\boldsymbol{H}=-\boldsymbol{\nabla}\cdot\boldsymbol{M}\quad\rightarrow\quad\mu_{\text{o}}\boldsymbol{\nabla}\cdot\boldsymbol{H}=\rho_{\text{bound}}^{(\text{m})}$.

b) In the above equations, $\rho_{\text{bound}}^{(\text{m})}=-\boldsymbol{\nabla}\cdot\boldsymbol{M}$ and $\boldsymbol{J}_{\text{bound}}^{(\text{m})}=\partial\boldsymbol{M}/\partial t-\varepsilon_{\text{o}}^{-1}\boldsymbol{\nabla}\times\boldsymbol{P}$. We may thus write

$$\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\text{bound}}^{(\text{m})}=\boldsymbol{\nabla}\cdot\left(\frac{\partial\boldsymbol{M}}{\partial t}-\varepsilon_{\text{o}}^{-1}\boldsymbol{\nabla}\times\boldsymbol{P}\right)=\frac{\partial(\boldsymbol{\nabla}\cdot\boldsymbol{M})}{\partial t}-\varepsilon_{\text{o}}^{-1}\overbrace{\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{P})}^{0}=\frac{\partial(\boldsymbol{\nabla}\cdot\boldsymbol{M})}{\partial t}=-\frac{\partial\rho_{\text{bound}}^{(\text{m})}}{\partial t}$$

$$\rightarrow\quad\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\text{bound}}^{(\text{m})}+\frac{\partial\rho_{\text{bound}}^{(\text{m})}}{\partial t}=0.$$