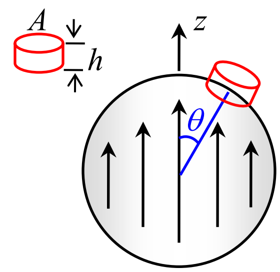
Problem 2.42) From Maxwell's 4[th] equation, we find the bound magnetic charge-density to be given by $\rho^{(m)}_{\text{bound}} = -\boldsymbol{\nabla}\cdot\boldsymbol{M}(\boldsymbol{r})$. Take a small pillbox and place it anywhere inside the sphere. The magnetization entering the box will be equal to that leaving the box and, therefore, the divergence of $\boldsymbol{M}(\boldsymbol{r}) = M_{\text{o}}\hat{\boldsymbol{z}}$ will be zero everywhere inside the sphere. The only points where the divergence will be non-zero are at the surface of the sphere. The figure shows a small, thin pillbox placed at $(r = R, \theta, \phi)$. Let A and h denote the base area and height of the pillbox, respectively; both A and h could be as small as desired. The flux of $\boldsymbol{M}$ entering from the bottom of the pillbox is $M_{\text{o}}A\cos\theta$, and this is the only contribution to the integral of $\boldsymbol{M}(\boldsymbol{r})$ over the pillbox surface, provided that h is much small than the pillbox diameter. The divergence of $\boldsymbol{M}$ at $(r = R, \theta, \phi)$ is thus given by $-M_{\text{o}}A\cos\theta/(Ah)$ in the limit of small A and h. Therefore, $\rho^{(m)}_{\text{bound}}(R, \theta, \phi) = M_{\text{o}}\cos\theta/h$. Since the charges are confined to the surface, we should use the surface-charge-density $\sigma^{(m)}_{\text{bound}} = h\rho^{(m)}_{\text{bound}}$ instead of the volume charge-density. Consequently, $\sigma^{(m)}_{\text{bound}}(R, \theta, \phi) = M_{\text{o}}\cos\theta$.



To determine the bound electric current-density $\boldsymbol{J}^{(e)}_{\text{bound}} = \mu_{\text{o}}^{-1}\boldsymbol{\nabla}\times\boldsymbol{M}(\boldsymbol{r})$, we use a small rectangular loop (length $= \ell$, width $= w$) at various locations within and on the surface of the sphere in order to calculate the curl of $\boldsymbol{M}$. For all locations within the sphere and for all orientations of the loop, the integral of $\boldsymbol{M}(\boldsymbol{r})$ around the loop turns out to be zero. When the loop is placed on the surface at $(r = R, \theta, \phi)$ and oriented perpendicular to $\hat{\boldsymbol{\phi}}$, as shown, the line integral on the lower leg of the loop will be nonzero $(\ell M_{\text{o}}\sin\theta)$. The curl of $\boldsymbol{M}(\boldsymbol{r})$ will then be nonzero, as the other legs do not contribute to the integral, provided that $w \ll \ell$. The curl will then be given by $[\ell M_{\text{o}}\sin\theta/(\ell w)]\hat{\boldsymbol{\phi}}$ in the limit when ℓ and w both tend to zero. The bound current-density at $(r = R, \theta, \phi)$ is thus given by $\boldsymbol{J}^{(e)}_{\text{bound}} = \mu_{\text{o}}^{-1}(M_{\text{o}}\sin\theta/w)\hat{\boldsymbol{\phi}}$. Since the current is confined to a thin layer on the surface, we could use the surface-current-density $\boldsymbol{J}^{(e)}_{\text{s-bound}} = w\boldsymbol{J}^{(e)}_{\text{bound}}$ instead of the bulk current-density. Consequently, $\boldsymbol{J}^{(e)}_{\text{s-bound}} = \mu_{\text{o}}^{-1}M_{\text{o}}\sin\theta\hat{\boldsymbol{\phi}}$.