Problem 2.38)

a) The magnetic flux through the circular loop is the integral of the B-field over the surface are of the loop. The field is uniform and confined to an area A defined by the pole-pieces; therefore, $\Phi(t) = AB_0 \cos(\omega t)$.

b) Using Stokes' theorem, Faraday's law, $\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$, may be written in integral form as follows:

$$\oint_{loop} \boldsymbol{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_{surface} \boldsymbol{B} \cdot d\boldsymbol{s} = -\frac{d\Phi(t)}{dt} = AB_0 \omega \sin(\omega t).$$

Symmetry dictates that the E-field be uniform around the circle and directed along the azimuthal axis $\hat{\boldsymbol{\varphi}}$. Therefore,

$$2\pi\rho E_\varphi = AB_0 \omega \sin(\omega t) \quad \rightarrow \quad \boldsymbol{E} = \frac{AB_0 \omega \sin(\omega t)}{2\pi\rho}\, \hat{\boldsymbol{\varphi}}.$$

c) The induced voltage in the loop is $V(t) = \oint \boldsymbol{E} \cdot d\boldsymbol{\ell} = AB_0 \omega \sin(\omega t)$. Considering that, in accordance with Ohm's law, $V(t) = R\, I(t)$, we will have $I(t) = (AB_0 \omega / R) \sin(\omega t)$.