Problem 2.37)

a) The polarization $\boldsymbol{P}$ has units of *coulomb*/m^2. Since the units of current are *ampere*, and since, by definition, electrical current I is the rate-of-flow of charge ΔQ during the time interval Δt, that is $I = \Delta Q/\Delta t$ in the limit when $\Delta t \to 0$, the units of charge are $coulomb = ampere \cdot sec$. Therefore, the units of $\boldsymbol{P}$ are $ampere \cdot sec/m^2$.

b) The magnetization $\boldsymbol{M}$ has units of $weber/m^2$. Since, by definition, $\boldsymbol{B} = \mu_0 \boldsymbol{H} + \boldsymbol{M}$, the units of $\boldsymbol{M}$ must be the same as those of $\boldsymbol{B}$. The Lorentz force law states that $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$, where $\boldsymbol{F}$ is force (units: *newton*), q is electrical charge (units: *coulomb*), $\boldsymbol{E}$ is electric field (units: *volt*/*m*), $\boldsymbol{V}$ is velocity (units: m/sec), and $\boldsymbol{B}$ is magnetic induction (units: $weber/m^2$). We may thus write: $newton = coulomb \cdot (m/sec) \cdot (weber/m^2)$. Now, from newton's law $\boldsymbol{F} = m\boldsymbol{a}$, we have $newton = kg \cdot m/sec^2$. Also, from $I = \Delta Q/\Delta t$, we have $coulomb = ampere \cdot sec$. Consequently,

$$weber/m^2 = (kg \cdot m/sec^2)/(ampere \cdot m) = kg/(ampere \cdot sec^2).$$

c) The electrical resistance R has units of *ohm*. According to Ohm's law, the voltage V and the current I of a resistor R are related via $V = RI$. Therefore, R has units of *volt*/*ampere*. From the Lorentz force law cited in part (b), we know that $newton = coulomb \cdot (volt/m)$, that is, $volt = newton \cdot m/coulomb = kg \cdot m^2/(ampere \cdot sec^3)$. Consequently, $ohm = kg \cdot m^2/(ampere^2 \cdot sec^3)$.

d) The units of capacitance C are *farad*. The relation between electrical charge on the plates of a capacitor and the voltage difference between those plates is $Q = CV$. Consequently,

$$farad = coulomb/volt = ampere \cdot sec/(newton \cdot m/coulomb) = ampere^2 \cdot \sec^4/(kg \cdot m^2).$$

e) The units of inductance L are *henry*. The relation between the voltage V and the current I of a solenoid are $V = L(dI/dt)$. Therefore, the units of L must be $volt \cdot sec/ampere$. We know from the Lorentz force law that $volt = newton \cdot m/coulomb$. Consequently,

$$henry = newton \cdot m \cdot sec/(coulomb \cdot ampere) = kg \cdot m^2/(ampere^2 \cdot sec^2).$$

f) The electromagnetic momentum-density $\boldsymbol{p}_{EM} = \boldsymbol{E} \times \boldsymbol{H}/c^2$ has units of the E-field times the units of the H-field divided by the units of velocity squared, i.e., $(volt/m) \cdot (ampere/m)/(m^2/sec^2)$. From the Lorentz force law we know that $volt/m = newton/coulomb = kg \cdot m/(sec^2 \cdot coulomb)$. Therefore, the units of $\boldsymbol{p}_{EM}$ are $kg/(m^2 \cdot sec)$.