Problem 2.36)

a) Electric field $\boldsymbol{E}$ $[volt/meter]$: Use Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$ and the Lorentz force law $\boldsymbol{F} = q\boldsymbol{E}$. The units of force $\boldsymbol{F}$ are thus $[kg \cdot meter/\mathrm{sec}^2]$, and the units of the electric field are $[kg \cdot meter/(coulomb \cdot \mathrm{sec}^2)]$ or $[kg \cdot meter/(ampere \cdot \mathrm{sec}^3)]$.

b) Magnetic induction $\boldsymbol{B}$ $[weber/m^2]$: Use Faraday's law, $\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$, in conjunction with the units of $\boldsymbol{E}$ determined in part (a). The curl operator does differentiation with respect to spatial coordinates; therefore, the units of $\boldsymbol{\nabla} \times \boldsymbol{E}$ are those of $\boldsymbol{E}$ divided by the length unit, $[meter]$. On the right-hand-side of the equation, the time-derivative of $\boldsymbol{B}$ has the units of $\boldsymbol{B}$ divided by those of time, $[second]$. Consequently, $[weber/m^2] = [kg/(ampere \cdot \mathrm{sec}^2)]$.

c) Poynting vector $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$ $[volt \cdot ampere/m^2]$: The units of $\boldsymbol{H}$ are $[ampere/meter]$, and the units of $\boldsymbol{E}$ were found in part (a) to be $[kg \cdot meter/(ampere \cdot \mathrm{sec}^3)]$. Multiplying the two, we find the units of $\boldsymbol{S}$ to be $[volt \cdot ampere/m^2] = [kg/sec^3]$. Note that $\boldsymbol{S}$ is expected to have the units of energy per unit area per unit time. Energy, however, has the units of force times displacement, that is, $[kg \cdot meter^2/sec^2]$. Dividing by $[meter^2 \cdot sec]$ then yields the units of S as before, namely, $[kg/sec^3]$.

d) Permittivity of free space ε_0 $[farad/meter]$: The charge Q and the voltage V of a capacitor are related via $Q = CV$. The capacitance C has units of $[farad]$, the units of Q are $[coulomb] = [ampere \cdot sec]$, and the units of V are $[kg \cdot meter^2/(ampere \cdot \mathrm{sec}^3)]$; see part (a). Consequently, the units of ε_0 are $[farad/meter] = [ampere^2 \cdot \mathrm{sec}^4/(kg \cdot meter^3)]$.

e) Permeability of free space μ_0 $[henry/meter]$: Use the fact that in free space $\boldsymbol{B} = \mu_0 \boldsymbol{H}$. The units of $\boldsymbol{H}$ are $[ampere/meter]$; those of $\boldsymbol{B}$ were found in part (b) to be $[kg/(ampere \cdot \mathrm{sec}^2)]$. Consequently, the units of μ_0 are $[henry/meter] = [kg \cdot meter/(ampere^2 \cdot \mathrm{sec}^2)]$.