Problem 2-30)

a) Ignoring the dynamic effects, the equation $\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}+\partial_t\boldsymbol{D}$ simplifies to $\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J}_{\text{free}}$.

$$\oint_{\text{loop}}\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{\ell}=I(t)\quad\rightarrow\quad 2\pi\rho H_\varphi(\rho,t)=I_0\sin(2\pi ft)\quad\rightarrow\quad H_\varphi(\rho,t)=(I_0/2\pi\rho)\sin(2\pi ft).$$

b) $\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial_t\boldsymbol{B}\quad\rightarrow\quad\boldsymbol{\nabla}\times[E_z(\rho,t)\hat{\boldsymbol{z}}]=-\mu_0\partial_t[H_\varphi(\rho,t)\hat{\boldsymbol{\varphi}}].$

$$\rightarrow\quad -[\partial E_z(\rho,t)/\partial\rho]\hat{\boldsymbol{\varphi}}=-\left(\frac{\mu_0 I_0}{2\pi\rho}\right)(2\pi f)\cos(2\pi ft)\,\hat{\boldsymbol{\varphi}}$$

$$\rightarrow\quad E_z(\rho,t)=\left(\frac{\mu_0 I_0}{2\pi}\right)(2\pi f)(\ln\rho)\cos(2\pi ft).$$

c) Considering that $\boldsymbol{J}_{\text{free}}=0$ for $\rho>0$, we will have $\boldsymbol{\nabla}\times\boldsymbol{H}=\partial_t\boldsymbol{D}$. Therefore,

$$\boldsymbol{\nabla}\times[H_\varphi(\rho,t)\hat{\boldsymbol{\varphi}}]=\varepsilon_0\partial_t[E_z(\rho,t)\hat{\boldsymbol{z}}]$$

$$\rightarrow\quad \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho H_\varphi(\rho,t)]\hat{\boldsymbol{z}}=-\left(\frac{\mu_0\varepsilon_0 I_0}{2\pi}\right)(2\pi f)^2(\ln\rho)\sin(2\pi ft)\,\hat{\boldsymbol{z}}$$

$$\rightarrow\quad \partial[\rho H_\varphi(\rho,t)]/\partial\rho=-\frac{I_0}{2\pi}(2\pi f/c)^2(\rho\ln\rho)\sin(2\pi ft)$$

$$\rightarrow\quad \rho H_\varphi(\rho,t)=-\frac{I_0}{4\pi}(2\pi f/c)^2\rho^2(\ln\rho-½)\sin(2\pi ft)$$

$$\rightarrow\quad H_\varphi(\rho,t)=-\frac{I_0}{4\pi}(2\pi f/c)^2\rho(\ln\rho-½)\sin(2\pi ft).$$

This term must be added to the expression for $H_\varphi(\rho,t)$ found in part (a) as a 1^{st}-order correction.

d) $\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial_t\boldsymbol{B}\quad\rightarrow\quad\boldsymbol{\nabla}\times[E_z(\rho,t)\hat{\boldsymbol{z}}]=\frac{\mu_0 I_0}{4\pi c^2}(2\pi f)^3\rho(\ln\rho-½)\cos(2\pi ft)\,\hat{\boldsymbol{\varphi}}$

$$\rightarrow\quad -[\partial E_z(\rho,t)/\partial\rho]\hat{\boldsymbol{\varphi}}=\frac{\sqrt{\mu_0/\varepsilon_0}\,I_0}{4\pi}(2\pi f/c)^3(\rho\ln\rho-½\rho)\cos(2\pi ft)\,\hat{\boldsymbol{\varphi}}$$

$$\rightarrow\quad E_z(\rho,t)=-\left(\frac{Z_0 I_0}{4\pi}\right)(2\pi f/c)^3(½\rho^2\ln\rho-¼\rho^2-¼\rho^2)\cos(2\pi ft)$$

$$\rightarrow\quad E_z(\rho,t)=\left(\frac{Z_0 I_0}{8\pi}\right)(2\pi f/c)^3\rho^2(1-\ln\rho)\cos(2\pi ft).$$

The above term must be added to the expression for $E_z(\rho,t)$ found in part (b) as the next correction term.