Problem 2-27) $V_0 = RI(t) + L\frac{\mathrm{d}I(t)}{\mathrm{d}t} \quad \rightarrow \quad I(t) = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right); \quad t \geq 0.$

In the above expression for $I(t)$, the initial conditions have been properly taken into account to ensure that, at $t = 0^+$ (i.e., immediately after the switch is closed), $I(0^+) = 0$. As for the induced voltage across the coil (or solenoid), we find

$$V_L(t) = L\frac{\mathrm{d}I(t)}{\mathrm{d}t} = V_0 e^{-Rt/L}; \qquad t \geq 0.$$

The total energy $\mathcal{E}$ delivered to the solenoid may now be computed, as follows:

$$\mathcal{E} = \int_{t=0}^{\infty} V_L(t)I(t)\mathrm{d}t = (V_0^2/R)\int_0^{\infty}\left(e^{-Rt/L} - e^{-2Rt/L}\right)\mathrm{d}t = \frac{V_0^2}{R}\left(\frac{L}{R} - \frac{L}{2R}\right) = ½L(V_0/R)^2$$
$$= ½L \times I^2(\infty).$$

In words, the total electrical energy delivered to the coil equals $½L$ times the square of the current in the steady-state, namely, $I(\infty) = V_0/R$.

Note that, initially, the battery supplies energy to both the coil and the resistor. As time goes on, however, the coil is gradually filled with magnetic energy and, eventually, stops receiving additional energy. The steady-state current $I(\infty)$ maintains the internal magnetic field of the coil, but no voltage is required to support this current [i.e., $V_L(\infty) = 0$]. In the end, all the incoming energy supplied by the battery goes into the resistor R, which converts this energy into heat. Needless to say, so long as the steady-state current $I(\infty)$ flows through the system, the magnetic field inside the coil remains constant, that is, the energy that was supplied by the battery in the early stages of the process (in the form of the magnetic field inside the coil) continues to reside within the coil. One may write the (stored) energy content of the coil in the following way:

$$\mathcal{E} = ½L \times I^2(\infty) = ½(\mu_0 N^2 A/\ell) \times I^2(\infty) = ½\mu_0[(N/\ell)I(\infty)]^2(A\ell).$$

In this equation, N/ℓ is the number of turns per unit-length and, therefore, $(N/\ell)I(\infty)$ is the surface current-density J_s of the coil, which equals the magnetic field H inside the solenoid. Given that $A\ell$ appearing on the right-hand side of the equation is the volume of the coil, the energy-density of the magnetic field stored within the solenoid is seen to be $½\mu_0 H^2$.