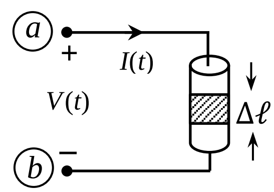
Problem 2-25) For electric circuits, we rely on a quasi-static treatment in which the various lumped circuit elements (e.g., resistors, capacitors, inductors, transistors, etc.) do *not* radiate electro-magnetic waves. It is only within this quasi-static regime that one can define a voltage difference (or potential drop) across an element as $V_a - V_b = \int_a^b \boldsymbol{E} \cdot d\boldsymbol{\ell}$, then propose that the sum of all voltage drops around a closed loop must be equal to zero (i.e., Kirchhoff's voltage rule). Obviously, if $\partial\boldsymbol{B}/\partial t \neq 0$ within the area that is enclosed by the loop, then from Maxwell's third equation, $\oint \boldsymbol{E} \cdot d\boldsymbol{\ell} \neq 0$. However, if the variations of $\boldsymbol{B}$ with time are slow enough (i.e., quasi-static approximation), then the Kirchhoff voltage rule stated above can be justified.

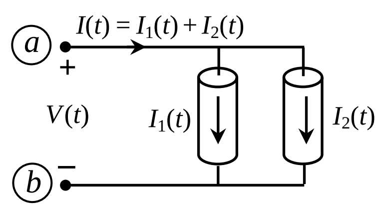
The Lorentz force law, $\boldsymbol{f} = q(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$, is the statement of the force exerted by $\boldsymbol{E}$ and $\boldsymbol{B}$ fields on a point-charge q moving with velocity $\boldsymbol{V}$. However, $\boldsymbol{V} \times \boldsymbol{B}$ is always perpendicular to the direction of motion ($\boldsymbol{V}$) and, as such, no work can be done by the B-field on a moving charge; the only work is done by the E-field. If the point-charge q moves a distance $\Delta\boldsymbol{\ell}$ under the influence of an electric field $\boldsymbol{E}$, then the work performed by the E-field on the charged particle will be $W = q\boldsymbol{E} \cdot \Delta\boldsymbol{\ell}$.



With reference to the simple resistor shown on the right, let ρ be the charge-density within the resistor, S the cross-sectional area, and $\Delta\ell$ the incremental length of the shaded segment. The charge content of the shaded area, $\Delta Q = \rho S \Delta\ell$, moves out of the shaded area in time Δt. Thus the current is given by $I(t) = \Delta Q/\Delta t = \rho S \Delta\ell/\Delta t$. The work done on the charge ΔQ during the time-interval Δt is $\Delta W = \Delta Q(\boldsymbol{E} \cdot \Delta\boldsymbol{\ell}) = I(t)\Delta t(\boldsymbol{E} \cdot \Delta\boldsymbol{\ell})$. Since $I(t)$ at any given time t is the same everywhere along the length of the resistor (Kirchhoff's current rule), the total work done by the E-field on the charges within the resistor may be written

$$W = \int_a^b dW = I(t)\Delta t \int_a^b \boldsymbol{E} \cdot d\boldsymbol{\ell} = I(t)V(t)\Delta t.$$

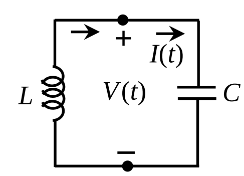
Work and energy are the same thing; therefore, the energy given to the resistor between t and $t + \Delta t$ must be $\mathcal{E}(t) = V(t)I(t)\Delta t$, and the electrical power delivered to the resistor is, therefore, $P(t) = \mathcal{E}(t)/\Delta t = V(t)I(t)$.



If the circuit happens to have multiple branches, then the energy must be computed for each branch and subsequently added up. In the circuit depicted on the right, both elements see the same voltage drop $V(t)$, but their respective currents are $I_1(t)$ and $I_2(t)$. The total electric power delivered to the circuit is then

$$P(t) = V(t)I_1(t) + V(t)I_2(t) = V(t)[I_1(t) + I_2(t)] = V(t)I(t).$$

If $V(t)I(t) > 0$, then energy is delivered to the circuit from the outside. In contrast, if $V(t)I(t) < 0$, then the circuit delivers energy to the outside world.



For an inductor, the same argument as used above for a resistor shows that the power delivered to (or received from) the inductor may be expressed as $P(t) = V(t)I(t)$. For a capacitor, however, the direct argument is difficult to make. Nevertheless, the parallel LC circuit shown on the right reveals that the energy going into C must come out of L, and vice-versa. Hence for a capacitor, we also have $P(t) = V(t)I(t)$.