**Problem 22)**

a)
$$\begin{aligned}
\mathrm{d}\mathcal{E}(t)/\mathrm{d}t &= \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{p}(t)/\mathrm{d}t = E_o \mathrm{d}p_z(t)/\mathrm{d}t = E_o \mathrm{d}[p(t)\cos\theta(t)]/\mathrm{d}t\\
&= E_o\cos\theta(t)[\mathrm{d}p(t)/\mathrm{d}t] - E_o\, p(t)\sin\theta(t)[\mathrm{d}\theta(t)/\mathrm{d}t]\\
&= E_o\cos\theta(t)[\mathrm{d}p(t)/\mathrm{d}t] + |\boldsymbol{p}(t)\times\boldsymbol{E}|[\mathrm{d}\theta(t)/\mathrm{d}t]\\
&= E_o\cos\theta(t)[\mathrm{d}p(t)/\mathrm{d}t] + I_o[\mathrm{d}^2\theta(t)/\mathrm{d}t^2][\mathrm{d}\theta(t)/\mathrm{d}t]\\
&= E_o\cos\theta(t)[\mathrm{d}p(t)/\mathrm{d}t] + (\mathrm{d}/\mathrm{d}t)\{\tfrac{1}{2} I_o[\mathrm{d}\theta(t)/\mathrm{d}t]^2\}
\end{aligned}$$

$\boxed{T_x(t)\text{, component of torque along } x\text{-axis}}$

The time-rate-of-change $\mathrm{d}\mathcal{E}(t)/\mathrm{d}t$ of the electromagnetic energy given to the dipole thus has two components. The first, associated with the changing dipole magnitude, $\mathrm{d}p(t)/\mathrm{d}t$, appears as the first term on the right-hand-side of the above equation. The second component, associated with the increase or decrease in the kinetic energy $I_o[\mathrm{d}\theta(t)/\mathrm{d}t]^2$ of the rotating dipole, constitutes the second term.

b) The electromagnetic energy exchanged with the dipole in consequence of its changing orientation angle $\theta(t)$ appears as either an increase or a decrease in the dipole's rotational kinetic energy. The energy exchanged with the field in the process of changing the dipole strength $p(t)$, however, causes either an increase or a decrease in the *internal* energy of the dipole. This energy may be stored inside the dipole (i.e., in a mass-and-spring model of the dipole, this would correspond to the potential energy of the spring or the kinetic energy of the mass attached to the spring), it could be fully or partially converted to heat, or it could be an altogether different kind of internal energy. Maxwell's equations do not prescribe the mechanism by which energy is taken in or given out by the dipole; they only express the rate of exchange of energy between the dipole and the electromagnetic field.

c) The energy given to the dipole comes from the $E$-field energy [density $= \tfrac{1}{2}\varepsilon_o E^2(\boldsymbol{r},t)$] that resides in the region surrounding the dipole. Similarly, the energy taken from the dipole goes into building up the surrounding $E$-field energy. The total $E$-field, of course, is the superposition of the external $E$-field and the dipolar $E$-field, namely, $E_o\hat{\boldsymbol{z}} + \boldsymbol{E}_{\text{dipole}}(\boldsymbol{r},t)$. It is the cross-term between these two fields in the expression of the $E$-field energy density, integrated over the entire space, that accounts for the energy given to or taken away from the dipole.