

Problem 14)

a) $\vec{\nabla} \cdot \vec{D} = \rho_{\text{free}} \Rightarrow \vec{D}_{\perp}(\vec{r}_0, t)$ is continuous, that is, $\vec{D}_{\perp}(\vec{r}_0^-, t) = \vec{D}_{\perp}(\vec{r}_0^+, t)$

Unless there is surface charge density $\sigma_{\text{free}}(\vec{r}_0, t)$ at the surface, in which case $\vec{D}_{\perp}(\vec{r}_0^+, t) - \vec{D}_{\perp}(\vec{r}_0^-, t) = \sigma_{\text{free}}(\vec{r}_0, t)$

b) $\vec{\nabla} \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \vec{H}_{\parallel}(\vec{r}_0, t)$ is continuous, that is, $\vec{H}_{\parallel}(\vec{r}_0^-, t) = \vec{H}_{\parallel}(\vec{r}_0^+, t)$

Unless there is surface current density $\vec{J}_s(\vec{r}_0, t)$ at the surface, in which case $\vec{H}_{\parallel}(\vec{r}_0^+, t) - \vec{H}_{\parallel}(\vec{r}_0^-, t)$ is equal in magnitude and perpendicular in direction to $\vec{J}_s(\vec{r}_0, t)$. Note that \vec{J}_s in general could have contributions from both \vec{J}_{free} and $\vec{J}_{\text{bound}} = \partial \vec{P}(\vec{r}_0, t) / \partial t$.

c) $\vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t \Rightarrow \vec{E}_{\parallel}(\vec{r}_0, t)$ is continuous, that is, $\vec{E}_{\parallel}(\vec{r}_0^-, t) = \vec{E}_{\parallel}(\vec{r}_0^+, t)$

Unless there is surface current density of magnetic monopoles, $\partial \vec{M} / \partial t$ at the surface, in which case $\vec{E}_{\parallel}(\vec{r}_0^+, t) - \vec{E}_{\parallel}(\vec{r}_0^-, t)$ is equal in magnitude and perpendicular in direction to the surface magnetic current density associated with $\partial \vec{M}(\vec{r}_0, t) / \partial t$.

d) $\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \vec{B}_{\perp}(\vec{r}_0, t)$ is continuous, that is, $\vec{B}_{\perp}(\vec{r}_0^-, t) = \vec{B}_{\perp}(\vec{r}_0^+, t)$.