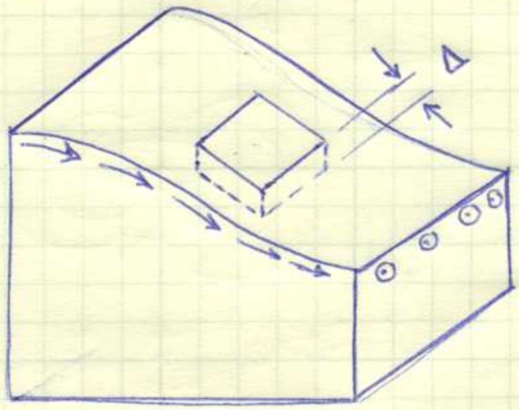
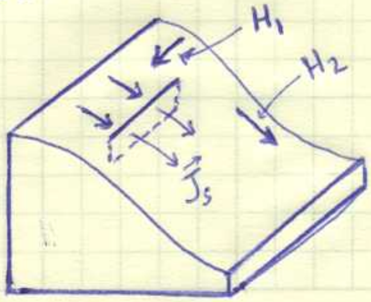
Problem 13) a) In general $\vec{J}$ has units of Amp/m^2. Therefore $\vec{J}_s$ being defined as "$\vec{J}$ times Δ" must have units of Amp/m.

b) Consider a small box of thickness Δ at the surface of the conductor. The top surface of the box is directly on the surface of the conductor, while its bottom surface is a distance



Δ below the conductor's surface. No current enters or exits the top and bottom surfaces of this box, because $\vec{J}(\vec{r})$ is everywhere parallel to the surface. The four surfaces on the sides of the box are the only places where the current can enter/exit. If we multiply $\vec{J}(\vec{r})$ by Δ to obtain $\vec{J}_s(\vec{r})$, then the divergence operator becomes a 2-dimensional operator that acts on the 4 sides of the square shown in the figure (rather that the 6 sides of the cube). The net current brought into the small cube causes a time rate of change of the volume charge density $\partial\rho(\vec{r})/\partial t$. But if $\rho(\vec{r})$ is multiplied by Δ it becomes the surface charge density, $\sigma(\vec{r})$. We may thus take the conservation of charge equation, $\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0$, and multiply it by Δ to obtain $\vec{\nabla}\cdot\vec{J}_s+\frac{\partial\sigma}{\partial t}=0$.

c) Consider a rectangle of length L and width Δ, placed at the surface and $\perp$ to the surface, as shown. If the



rectangle is $\perp$ to the direction of current $\vec{J}_s$, then the total current crossing the rectangle will be $J(\vec{r})\, L\Delta = J_s(\vec{r})\, L$. This must be equal to the line integral of $\vec{H}$ around the rectangle. Since Δ is assumed to be small ($\Delta \to 0$), we may ignore the contribution of the line integral along the narrow dimension of the rectangle. Denoting the component of the $\vec{H}$-field that is parallel to the surface and $\perp$ to $\vec{J}_s$ by H_1 (See the figure), we'll have: $\oint \vec{H} \cdot d\vec{\ell} = J_s(\vec{r})\, L \Rightarrow \left[H_1^{(above)} - H_1^{(below)} \right] L = J_s(\vec{r})\, L$.

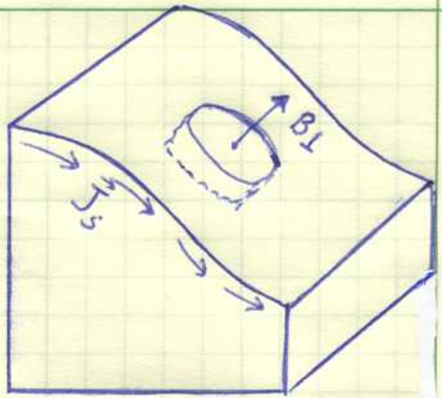
$$\Rightarrow H_1^{(above)} - H_1^{(below)} = J_s(\vec{r}).$$

"above" and "below" refer to the points immediately above and immediately below the current-carrying surface at the location $\vec{r}$.

We thus see that the component $\vec{H}_1$ of the $\vec{H}$-field that is parallel to the surface <u>and</u> perpendicular to the direction of the surface current $\vec{J}_s$ must be <u>discontinuous</u> at the surface. The magnitude of this discontinuity is $J_s(\vec{r})$.

We can repeat the same argument for the component $\vec{H}_2$ of the field (See the figure). $\vec{H}_2$ is $\parallel$ to the surface <u>and</u> $\parallel$ to $\vec{J}_s$. This time, however, the rectangle must be parallel to $\vec{H}_2$ (still $\perp$ to the surface). Since <u>no</u> current crosses such a rectangle, the $\oint \vec{H} \cdot d\vec{\ell}$ must be zero; that is, $H_2^{(above)} = H_2^{(below)}$. In words, the component of the $\vec{H}$-field that is parallel to surface <u>and</u> parallel to $\vec{J}_s$ must be <u>continuous</u> at the surface.

d) Choose a thin pillbox at the surface of the conductor. The side walls can be ignored, because the pillbox can be as thin as desired. Since $\vec{\nabla}\cdot\vec{B}=0$, the contributions of the top and bottom facets of the pillbox must cancel out. Therefore, $B_\perp^{(above)} = B_\perp^{(below)}$.



In words, $B_\perp$ is continuous at the surface.