
Problem 12) The pill-box's top and bottom surface areas are Δs . The pill-box can be made as thin as desired, so the side-walls may be assumed to contribute negligibly to the overall surface integral. On the top surface of the box, the surface-normal (pointing outward) is parallel to \mathbf{N} , so the top surface-integral is $(\mathbf{E}_1 \cdot \mathbf{N})\Delta s = E_1^\perp \Delta s$. On the bottom surface, the surface-normal (again pointing outward) is anti-parallel to \mathbf{N} ; therefore, the bottom surface-integral is $-(\mathbf{E}_2 \cdot \mathbf{N})\Delta s = -E_2^\perp \Delta s$. Consequently, the net integral of \mathbf{E} is given by $\oint \mathbf{E} \cdot d\mathbf{s} = (E_1^\perp - E_2^\perp)\Delta s$. Since the total charge enclosed by the pill-box is $Q = \sigma(\mathbf{r})\Delta s$, Gauss's law yields $\sigma(\mathbf{r}) = \epsilon_0(E_1^\perp - E_2^\perp)$.
