Problem 2.8)

a) $$\boldsymbol{B}(\boldsymbol{r},t)=\mu_0[\boldsymbol{H}_1(\boldsymbol{r},t)+\boldsymbol{H}_2(\boldsymbol{r},t)]=\frac{\mu_0 I_0}{2\pi}\left[\frac{1}{(d/2)+x}+\frac{1}{(d/2)-x}\right]\sin(2\pi f t)\,\hat{\boldsymbol{z}}.$$

Here x is the distance from the center-line. The expression can be slightly simplified to yield

$$\boldsymbol{B}(\boldsymbol{r},t)=\frac{2\mu_0 I_0 d}{\pi(d^2-4x^2)}\sin(2\pi f t)\,\hat{\boldsymbol{z}};\qquad |x|\le(d/2)-R.$$

b) The two wires contribute equally to the flux, so we must double the contribution of one wire to find the total flux, that is,

$$\Phi(t)=2\int_R^{d-R}\frac{\mu_0 I_0}{2\pi r}\sin(2\pi f t)\,\mathrm{d}r=\left(\frac{\mu_0 I_0}{\pi}\right)\sin(2\pi f t)\ln r\big|_R^{d-R}$$
$$=\left(\frac{\mu_0 I_0}{\pi}\right)\ln\left(\frac{d}{R}-1\right)\sin(2\pi f t).$$

c) $$\Phi(t)=L\,I(t)\quad\rightarrow\quad L=\left(\frac{\mu_0}{\pi}\right)\ln\left(\frac{d}{R}-1\right).$$

Note that both μ_0 and L have units of henry/meter.

d) Maxwell's third equation, $\boldsymbol{\nabla}\times\boldsymbol{E}=-\partial\boldsymbol{B}/\partial t$, also known as Faraday's law, can be written in integral form as $\oint\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{\ell}=-\mathrm{d}\Phi/\mathrm{d}t$, where the B-field flux is given by $\Phi(t)=\int_{\text{surface}}\boldsymbol{B}(\boldsymbol{r},t)\cdot\mathbf{d}\boldsymbol{s}$. Taking the E-field integral around a rectangular loop, and noting that the legs of the rectangle that are aligned with the wires contribute equally to the integral, whereas contributions of the legs that are perpendicular to the wires cancel out, we will have

$$\oint\boldsymbol{E}\cdot\mathbf{d}\boldsymbol{\ell}=-\frac{\mathrm{d}\Phi}{\mathrm{d}t}\quad\rightarrow\quad E(t)=-\frac{1}{2}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}t}\right)=-\mu_0 I_0 f\ln\left(\frac{d}{R}-1\right)\cos(2\pi f t).$$