

Problem 7) Charge-current continuity equation: $\nabla \cdot \mathbf{J} + \partial\rho/\partial t = 0$.

For Box 1, the incoming and outgoing \mathbf{J} are equal, therefore, $\nabla \cdot \mathbf{J} = 0$, resulting in $\partial\rho/\partial t = 0$. Assuming that the wire has been without charge initially, i.e., $\rho(-\infty) = 0$, we conclude that $\rho(t) = 0$ along the entire length of the wire.

For Box 2, the current comes in from below, but does not leave the box. Therefore, $\oint \mathbf{J} \cdot d\mathbf{s} = -dQ_1/dt \rightarrow dQ_1/dt = J_0 s \sin(2\pi ft)$, where s is the cross-sectional area of the wire. For the accumulated charge at the top of the wire, we thus have $Q_1(t) = -\frac{J_0 s}{2\pi f} \cos(2\pi ft)$.

At the bottom of the wire, $Q_2(t) = +\frac{J_0 s}{2\pi f} \cos(2\pi ft)$. The dipole moment $\mathbf{p}(t) = Q_1(t) \cdot d \cdot \hat{\mathbf{z}}$ of the wire (length = d) is now given by $\mathbf{p}(t) = -[J_0 s d / (2\pi f)] \cos(2\pi ft) \hat{\mathbf{z}}$.

