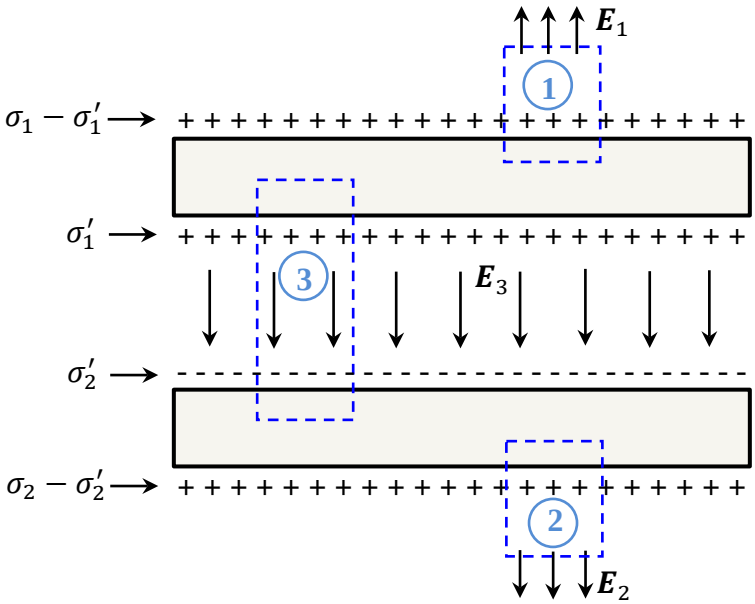
Problem 2-6) The net flux of the E-field through Box 3 is zero, because its top and bottom facets are inside the metal plates, where $\boldsymbol{E} = 0$. Therefore, $\sigma_1' + \sigma_2' = 0$.



The total charge-density on the two plates is $\sigma_1 + \sigma_2$. This creates equal fields E_1 and E_2 above both plates and below both plates, namely,

$$E_1 = E_2 = \tfrac{1}{2}\,(\sigma_1 + \sigma_2)/\varepsilon_0.$$

Now, consideration of Box 1 yields

$$E_1 = (\sigma_1 - \sigma_1')/\varepsilon_0,$$

and consideration of Box 2 yields

$$E_2 = (\sigma_2 - \sigma_2')/\varepsilon_0.$$

Therefore,

$$\sigma_1 - \sigma_1' = \sigma_2 - \sigma_2' \quad \rightarrow \quad \sigma_1' - \sigma_2' = \sigma_1 - \sigma_2 \quad \rightarrow \quad \begin{cases} \sigma_1' = \tfrac{1}{2}(\sigma_1 - \sigma_2) \\ \sigma_2' = \tfrac{1}{2}(\sigma_2 - \sigma_1) \end{cases}.$$

On the top facet of the upper plate, and also on the bottom facet of the lower plate, we have

$$\sigma_1 - \sigma_1' = \sigma_2 - \sigma_2' = \tfrac{1}{2}(\sigma_1 + \sigma_2).$$

The field E_3 in the region between the plates is given by $E_3 = \sigma_1'/\varepsilon_0 = \tfrac{1}{2}\,(\sigma_1 - \sigma_2)/\varepsilon_0$.