Problem 3)

$$\rho(\vec{r},t) = q\,\delta(x-f(t))\,\delta(y-g(t))\,\delta(z-h(t)) \quad \checkmark$$

$$\text{Velocity } \vec{v} = f'(t)\,\hat{x} + g'(t)\,\hat{y} + h'(t)\,\hat{z}$$

$$\vec{J}(\vec{r},t) = \rho(\vec{r},t)\,\vec{v}(\vec{r},t) = \left[f'(t)\,\hat{x} + g'(t)\,\hat{y} + h'(t)\,\hat{z}\right] q\,\delta(x-f(t))\,\delta(y-g(t))\,\delta(z-h(t)) \quad \checkmark$$

$$\vec{\nabla}\cdot\vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = q f'(t)\,\delta'(x-f)\,\delta(y-g)\,\delta(z-h) + q g'(t)\,\delta(x-f)\,\delta'(x-g)\,\delta(z-h)$$
$$+\, q h'(t)\,\delta(x-f)\,\delta(y-g)\,\delta'(z-h)$$

$$\frac{\partial \rho(\vec{r},t)}{\partial t} = q\frac{\partial}{\partial t}\left\{\delta(x-f)\,\delta(y-g)\,\delta(z-h)\right\} = q\left\{\frac{\partial}{\partial t}\delta(x-f(t))\right\}\delta(y-g)\,\delta(z-h) +$$
$$q\,\delta(x-f)\left\{\frac{\partial}{\partial t}\delta(y-g(t))\right\}\delta(z-h) + q\,\delta(x-f)\,\delta(y-g)\left\{\frac{\partial}{\partial t}\delta(z-h(t))\right\}$$

$$= -q f'(t)\,\delta'(x-f)\,\delta(y-g)\,\delta(z-h) - q g'(t)\,\delta(x-f)\,\delta'(y-g)\,\delta(z-h)$$
$$-\, q h'(t)\,\delta(x-f)\,\delta(y-g)\,\delta'(z-h)$$

Clearly $\vec{\nabla}\cdot\vec{J}(\vec{r},t) + \frac{\partial}{\partial t}\rho(\vec{r},t) = 0 \quad \checkmark$