
Problem 2)

a) $\rho_{\text{total}}(\mathbf{r}, t) = \rho_1(\mathbf{r}, t) + \rho_2(\mathbf{r}, t)$.

b) $\mathbf{J}_{\text{total}}(\mathbf{r}, t) = \rho_1(\mathbf{r}, t)\mathbf{V}_1(\mathbf{r}, t) + \rho_2(\mathbf{r}, t)\mathbf{V}_2(\mathbf{r}, t)$.

c) No. The continuity equation is based on the principle of conservation of charge. Since *individual* species 1 and 2 in this example are *not* conserved, the continuity equation does *not* apply to them as separate entities. For example, the change of charge density with time, $\partial\rho_1/\partial t$ or $\partial\rho_2/\partial t$, may have nothing to do with the inflow or outflow of charge produced by the local $\nabla\cdot\mathbf{J}_1(\mathbf{r}, t)$ or $\nabla\cdot\mathbf{J}_2(\mathbf{r}, t)$. Rather, the change of charge density may be caused by recombination of the two species.

d) Yes, the combined system satisfies the continuity equation $\nabla\cdot\mathbf{J}_{\text{total}}(\mathbf{r}, t) + \partial\rho_{\text{total}}(\mathbf{r}, t)/\partial t = 0$, because the total charge is a conserved entity. When, for example, Na^+ and Cl^- ions in a salt solution combine, we lose one unit of positive charge *and* one unit of negative charge, but the total amount of charge in the system remains the same.
