Problem 1.43)

a)

$$\begin{aligned}\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\psi) &= \boldsymbol{\nabla}\times\{\mathrm{i}\boldsymbol{k}_1\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]\}\\ &= \mathrm{i}^2\underbrace{(\boldsymbol{k}_1\times\boldsymbol{k}_1)}_{0}\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)] = 0.\end{aligned}$$

b)

$$\begin{aligned}\boldsymbol{\nabla}\cdot(\psi\boldsymbol{A}) &= \boldsymbol{\nabla}\cdot(\psi_0\boldsymbol{A}_0\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}-(\omega_1+\omega_2)t]\})\\ &= \mathrm{i}\psi_0(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{A}_0\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}-(\omega_1+\omega_2)t]\}\\ &= \{\mathrm{i}\boldsymbol{k}_1\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]\}\cdot\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)]\\ &\quad+\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]\,\{\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)]\}\\ &= (\boldsymbol{\nabla}\psi)\cdot\boldsymbol{A}+\psi(\boldsymbol{\nabla}\cdot\boldsymbol{A}).\end{aligned}$$

c)

$$\begin{aligned}\boldsymbol{\nabla}\times(\psi\boldsymbol{B}) &= \boldsymbol{\nabla}\times(\psi_0\boldsymbol{B}_0\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_1+\omega_3)t]\})\\ &= \mathrm{i}\psi_0(\boldsymbol{k}_1+\boldsymbol{k}_3)\times\boldsymbol{B}_0\exp\{\mathrm{i}[(\boldsymbol{k}_1+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_1+\omega_3)t]\}\\ &= \{\mathrm{i}\boldsymbol{k}_1\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]\}\times\boldsymbol{B}_0\exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)]\\ &\quad+\psi_0\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]\,\{\mathrm{i}\boldsymbol{k}_3\times\boldsymbol{B}_0\exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)]\}\\ &= (\boldsymbol{\nabla}\psi)\times\boldsymbol{B}+\psi\boldsymbol{\nabla}\times\boldsymbol{B}.\end{aligned}$$

d)

$$\begin{aligned}\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B}) &= \boldsymbol{\nabla}\cdot(\boldsymbol{A}_0\times\boldsymbol{B}_0\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\})\\ &= \mathrm{i}(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot(\boldsymbol{A}_0\times\boldsymbol{B}_0)\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\}\\ &= \mathrm{i}\boldsymbol{k}_2\cdot(\boldsymbol{A}_0\times\boldsymbol{B}_0)\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\}\\ &\quad+\mathrm{i}\boldsymbol{k}_3\cdot(\boldsymbol{A}_0\times\boldsymbol{B}_0)\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\}\\ &= (\mathrm{i}\boldsymbol{k}_2\times\boldsymbol{A}_0)\cdot\boldsymbol{B}_0\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\}\\ &\quad-(\mathrm{i}\boldsymbol{k}_3\times\boldsymbol{B}_0)\cdot\boldsymbol{A}_0\exp\{\mathrm{i}[(\boldsymbol{k}_2+\boldsymbol{k}_3)\cdot\boldsymbol{r}-(\omega_2+\omega_3)t]\}\\ &= \{\mathrm{i}\boldsymbol{k}_2\times\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)]\}\cdot\boldsymbol{B}_0\exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)]\\ &\quad-\{\mathrm{i}\boldsymbol{k}_3\times\boldsymbol{B}_0\exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)]\}\cdot\boldsymbol{A}_0\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)]\\ &= (\boldsymbol{\nabla}\times\boldsymbol{A})\cdot\boldsymbol{B}-(\boldsymbol{\nabla}\times\boldsymbol{B})\cdot\boldsymbol{A} = \boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A})-\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B}).\end{aligned}$$