Problem 1.42)

a) $\boldsymbol{\alpha}\cdot\boldsymbol{\beta} = (\boldsymbol{\alpha}' + \mathrm{i}\boldsymbol{\alpha}'')\cdot(\boldsymbol{\beta}' + \mathrm{i}\boldsymbol{\beta}'') = \overbrace{(\boldsymbol{\alpha}'\cdot\boldsymbol{\beta}' - \boldsymbol{\alpha}''\cdot\boldsymbol{\beta}'')}^{\text{Real}} + \mathrm{i}\overbrace{(\boldsymbol{\alpha}'\cdot\boldsymbol{\beta}'' + \boldsymbol{\alpha}''\cdot\boldsymbol{\beta}')}^{\text{Imaginary}}.$

b) $\boldsymbol{\alpha}\times\boldsymbol{\beta} = (\boldsymbol{\alpha}' + \mathrm{i}\boldsymbol{\alpha}'')\times(\boldsymbol{\beta}' + \mathrm{i}\boldsymbol{\beta}'') = (\boldsymbol{\alpha}'\times\boldsymbol{\beta}' - \boldsymbol{\alpha}''\times\boldsymbol{\beta}'') + \mathrm{i}(\boldsymbol{\alpha}'\times\boldsymbol{\beta}'' + \boldsymbol{\alpha}''\times\boldsymbol{\beta}').$

c) $\boldsymbol{\gamma}\times\boldsymbol{\gamma} = (\boldsymbol{\gamma}' + \mathrm{i}\boldsymbol{\gamma}'')\times(\boldsymbol{\gamma}' + \mathrm{i}\boldsymbol{\gamma}'') = (\cancelto{0}{\boldsymbol{\gamma}'\times\boldsymbol{\gamma}'} - \cancelto{0}{\boldsymbol{\gamma}''\times\boldsymbol{\gamma}''}) + \mathrm{i}(\boldsymbol{\gamma}'\times\boldsymbol{\gamma}'' + \underbrace{\cancel{\boldsymbol{\gamma}''\times\boldsymbol{\gamma}'}}_{-\boldsymbol{\gamma}'\times\boldsymbol{\gamma}''}) = 0 + \mathrm{i}0.$

d) $\boldsymbol{\alpha}\cdot(\boldsymbol{\beta}\times\boldsymbol{\gamma}) = (\boldsymbol{\alpha}' + \mathrm{i}\boldsymbol{\alpha}'')\cdot[(\boldsymbol{\beta}' + \mathrm{i}\boldsymbol{\beta}'')\times(\boldsymbol{\gamma}' + \mathrm{i}\boldsymbol{\gamma}'')]$

$$= (\boldsymbol{\alpha}' + \mathrm{i}\boldsymbol{\alpha}'')\cdot[(\boldsymbol{\beta}'\times\boldsymbol{\gamma}' - \boldsymbol{\beta}''\times\boldsymbol{\gamma}'') + \mathrm{i}(\boldsymbol{\beta}'\times\boldsymbol{\gamma}'' + \boldsymbol{\beta}''\times\boldsymbol{\gamma}')]$$

$$= [\boldsymbol{\alpha}'\cdot(\boldsymbol{\beta}'\times\boldsymbol{\gamma}' - \boldsymbol{\beta}''\times\boldsymbol{\gamma}'') - \boldsymbol{\alpha}''\cdot(\boldsymbol{\beta}'\times\boldsymbol{\gamma}'' + \boldsymbol{\beta}''\times\boldsymbol{\gamma}')] \longleftarrow \text{Real}$$

$$+\mathrm{i}[\boldsymbol{\alpha}'\cdot(\boldsymbol{\beta}'\times\boldsymbol{\gamma}'' + \boldsymbol{\beta}''\times\boldsymbol{\gamma}') + \boldsymbol{\alpha}''\cdot(\boldsymbol{\beta}'\times\boldsymbol{\gamma}' - \boldsymbol{\beta}''\times\boldsymbol{\gamma}'')]. \longleftarrow \text{Imaginary}$$