
Problem 1.42)

$$\begin{aligned}
 \text{a) } \alpha \cdot \beta &= (\alpha' + i\alpha'') \cdot (\beta' + i\beta'') = \overbrace{(\alpha' \cdot \beta' - \alpha'' \cdot \beta'')}^{\text{Real}} + i \overbrace{(\alpha' \cdot \beta'' + \alpha'' \cdot \beta')}^{\text{Imaginary}}. \\
 \text{b) } \alpha \times \beta &= (\alpha' + i\alpha'') \times (\beta' + i\beta'') = (\alpha' \times \beta' - \alpha'' \times \beta'') + i(\alpha' \times \beta'' + \alpha'' \times \beta'). \\
 \text{c) } \gamma \times \gamma &= (\gamma' + i\gamma'') \times (\gamma' + i\gamma'') = \underbrace{(\cancel{\gamma' \times \gamma'}^0 - \cancel{\gamma'' \times \gamma''}^0)}_{\text{Real}} + i(\gamma' \times \gamma'' + \underbrace{\gamma'' \times \gamma'}_{-\gamma' \times \gamma''}) = 0 + i0. \\
 \text{d) } \alpha \cdot (\beta \times \gamma) &= (\alpha' + i\alpha'') \cdot [(\beta' + i\beta'') \times (\gamma' + i\gamma'')] \\
 &= (\alpha' + i\alpha'') \cdot [(\beta' \times \gamma' - \beta'' \times \gamma'') + i(\beta' \times \gamma'' + \beta'' \times \gamma')] \\
 &= [\alpha' \cdot (\beta' \times \gamma' - \beta'' \times \gamma'') - \alpha'' \cdot (\beta' \times \gamma'' + \beta'' \times \gamma')] \leftarrow \text{Real} \\
 &\quad + i[\alpha' \cdot (\beta' \times \gamma'' + \beta'' \times \gamma') + \alpha'' \cdot (\beta' \times \gamma' - \beta'' \times \gamma'')]. \leftarrow \text{Imaginary}
 \end{aligned}$$
