

Problem 1-39) From Problem 35, part (b), we have $\nabla \times (\psi \mathbf{A}) = (\nabla \psi) \times \mathbf{A} + \psi \nabla \times \mathbf{A}$. Replacing \mathbf{A} , which, in general, is a function of position \mathbf{r} , with the constant vector \mathbf{C} , makes the curl term on the right-hand side of the above identity to vanish. We will then have $\nabla \times (\psi \mathbf{C}) = (\nabla \psi) \times \mathbf{C}$. Applying the Stokes theorem to the left-hand-side of the above equation, we find

$$\int_{\text{surface}} \nabla \times (\psi \mathbf{C}) \cdot d\mathbf{s} = \oint_{\text{boundary}} \psi(\mathbf{r}) \mathbf{C} \cdot d\boldsymbol{\ell} = \mathbf{C} \cdot \oint_{\text{boundary}} \psi(\mathbf{r}) d\boldsymbol{\ell}. \quad (1)$$

On the right-hand-side, the surface integral may be somewhat simplified, as follows:

$$\int_{\text{surface}} [(\nabla \psi) \times \mathbf{C}] \cdot d\mathbf{s} = - \int_{\text{surface}} [(\nabla \psi) \times d\mathbf{s}] \cdot \mathbf{C} = - \mathbf{C} \cdot \int_{\text{surface}} \nabla \psi(\mathbf{r}) \times d\mathbf{s}. \quad (2)$$

The above expressions are valid for *any* (arbitrary) constant vector \mathbf{C} and, moreover, they are equal to each other. We conclude that the coefficients of \mathbf{C} on the right-hand-sides of Eqs. (1) and (2) must be the same, that is, $\int_{\text{surface}} \nabla \psi(\mathbf{r}) \times d\mathbf{s} = - \oint_{\text{boundary}} \psi(\mathbf{r}) d\boldsymbol{\ell}$.
