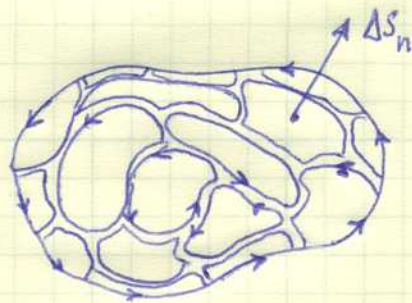


Problem 33)

Break up the surface S into a collection of small surfaces, $\Delta S_1, \Delta S_2, \dots$, with shared boundaries that cover the entire surface S . For each such small surface we have,



$$\int_{\Delta S_n} \vec{B} \cdot d\vec{s} \cong (\vec{\nabla} \times \vec{A}) \cdot \vec{\Delta S}_n = \frac{\oint \vec{A} \cdot d\vec{\ell}}{\Delta S_n} \Delta S_n = \oint_{\text{(Boundary of } \Delta S_n)} \vec{A} \cdot d\vec{\ell}$$

When the contributions of all the small surfaces are added up, the line-integrals on adjacent (shared) boundaries will cancel out.

What is left is the line-integral around the closed loop C that bounds the surface S . Therefore,

$$\int_S \vec{B} \cdot d\vec{s} \cong \sum_n \int_{\Delta S_n} \vec{B} \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{\ell} \quad \leftarrow \text{Stokes's theorem}$$

When the surface S is closed, its boundary C shrinks to zero. Then $\oint_C \vec{A} \cdot d\vec{\ell} = 0 \Rightarrow \oint_{\text{(closed surface)}} \vec{B} \cdot d\vec{s} = 0$. When the volume enclosed becomes small, $\vec{\nabla} \cdot \vec{B} = 0$.