Problem 1-31)

a) The dot-product $\boldsymbol{r}\cdot\boldsymbol{s}$ may be written in two different but equivalent ways, as follows:

$$\boldsymbol{r}\cdot\boldsymbol{s} = (x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}})\cdot\left(s_x\hat{\boldsymbol{x}} + s_y\hat{\boldsymbol{y}} + s_z\hat{\boldsymbol{z}}\right) = xs_x + ys_y + zs_z,$$

$$\boldsymbol{r}\cdot\boldsymbol{s} = |\boldsymbol{r}|\underbrace{\cancel{|\boldsymbol{s}|}}_{1}\cos\theta = |\boldsymbol{r}|\cos\theta = d.$$

b) In a plane perpendicular to $\boldsymbol{s}$, we have $xs_x + ys_y + zs_z = d$. Therefore, for all points in this plane, we may write

$$f(x,y,z,t) = \exp\left[\mathrm{i}2\pi\left(\frac{d}{\lambda} - \nu t\right)\right].$$

In each such plane, located at a fixed distance d from the origin, we will have

$$f(x,y,z,t) = \exp(\mathrm{i}2\pi d/\lambda)\exp(-\mathrm{i}2\pi\nu t).$$

$f(x,y,z,t)$ is thus an oscillatory function of time with frequency ν. The phase-factor $\exp(\mathrm{i}2\pi d/\lambda)$ is constant within each plane, but changes with increasing or decreasing distance d. However, if d is changed by an integer-multiple of λ, the phase-factor $\exp[\mathrm{i}2\pi(d+n\lambda)/\lambda] = \exp(\mathrm{i}2\pi d/\lambda)$ would remain the same. This is the proof that the wavelength of the plane-wave is equal to λ.

For a given point on a (plane) wave-front, in order to maintain a fixed phase ϕ_0 at all times t, the following relations must hold:

$$\exp\left[\mathrm{i}2\pi\left(\frac{d}{\lambda} - \nu t\right)\right] = \exp(\mathrm{i}\phi_0) \quad\rightarrow\quad 2\pi\left(\frac{d}{\lambda} - \nu t\right) = \phi_0 \quad\rightarrow\quad d = (\lambda\nu)t + (\lambda\phi_0/2\pi).$$

Clearly, the wavefront must be propagating at a speed $V = \lambda\nu$.