Problem 28)

$$e^{i\theta}+e^{2i\theta}+\cdots+e^{in\theta}=x(1+x+x^2+\cdots+x^{n-1}) \leftarrow x=e^{i\theta}$$

$$=x\,\frac{x^n-1}{x-1} \leftarrow \text{geometric Series}$$

$$=e^{i\theta}\,\frac{e^{in\theta}-1}{e^{i\theta}-1}=e^{i\theta}\,\frac{e^{in\theta/2}\left(e^{in\theta/2}-e^{-in\theta/2}\right)}{e^{i\theta/2}\left(e^{i\theta/2}-e^{-i\theta/2}\right)}$$

$$=e^{i(n+1)\theta/2}\,\frac{\sin(n\theta/2)}{\sin(\theta/2)}$$

Therefore,

$$(\cos\theta+\cos 2\theta+\cdots+\cos n\theta)+i(\sin\theta+\sin 2\theta+\cdots+\sin n\theta)=$$

$$\frac{\sin(n\theta/2)}{\sin(\theta/2)}\left[\cos(n+1)\theta/2+i\sin(n+1)\theta/2\right]$$

$$\Rightarrow \cos\theta+\cos 2\theta+\cdots+\cos n\theta=\frac{\sin(n\theta/2)\cos(n+1)\theta/2}{\sin(\theta/2)},$$

$$\sin\theta+\sin 2\theta+\cdots+\sin n\theta=\frac{\sin(n\theta/2)\sin(n+1)\theta/2}{\sin(\theta/2)}.$$

Consequently: $\dfrac{\sin\theta+\sin 2\theta+\cdots+\sin n\theta}{\cos\theta+\cos 2\theta+\cdots+\cos n\theta}=\tan\left[\dfrac{(n+1)\theta}{2}\right]$. ✓