

Problem 20)
$$\frac{df[g(x)]}{dx} = \frac{f[g(x+\Delta x)] - f[g(x)]}{\Delta x \rightarrow 0} = \frac{g(x+\Delta x) - g(x)}{\Delta x \rightarrow 0} \frac{f[g(x+\Delta x)] - f[g(x)]}{g(x+\Delta x) - g(x)}$$

$$= g'(x) \frac{f(u+\Delta u) - f(u)}{\Delta u \rightarrow 0} = g'(x) f'(u) = g'(x) f'[g(x)]. \checkmark$$

$$\frac{d}{dx} \sin(\cos x) = -\sin(x) \cos(\cos x). \checkmark$$

$$\frac{d}{dx} \exp(-kx^2) = -2kx \exp(-kx^2). \checkmark$$

$$\frac{d}{dx} \tan \sqrt{ax^2 + bx + c} = \frac{2ax + b}{2\sqrt{ax^2 + bx + c}} (1 + \tan^2 \sqrt{ax^2 + bx + c}). \checkmark$$

$$\frac{d}{dx} \ln(\tan^2 x) = 2 \tan(x) (1 + \tan^2 x) \frac{1}{\tan^2 x} = \frac{2(1 + \tan^2 x)}{\tan x} = \frac{2}{\sin x \cos x}. \checkmark$$

$$\frac{d}{dx} \tanh(\ln x) = \frac{1}{x} [1 - \tanh^2(\ln x)]. \checkmark$$

$$\frac{d}{dx} (x^x) = \frac{d}{dx} e^{x \ln x} = \frac{d}{dx} (x \ln x) e^{x \ln x} = (\ln x + 1) x^x. \checkmark$$

$$\frac{d}{dx} [\sinh^x(x)] = \frac{d}{dx} e^{x \ln[\sinh(x)]} = \frac{d}{dx} [x \ln(\sinh x)] e^{x \ln(\sinh x)}$$

$$= \left[\ln(\sinh x) + \frac{x \cosh x}{\sinh x} \right] \sinh^x(x).$$