
Problem 19) This problem may be solved by completing the square, as follows:

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0 \quad \rightarrow \quad x + \frac{b}{2a} = \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$
$$\rightarrow \quad x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Note that, since the coefficients a , b , and c of the quadratic equation are complex-valued, the square-root must be evaluated in the complex plane. However, the \pm sign in the above expressions is appropriate because the two roots of the complex number $(b^2 - 4ac)$ differ from each other by a phase angle of π , which results in the coefficient $\exp(i\pi) = -1$.
