Opti 501 **Solutions**

Problem 18) Method 1: Define $\boldsymbol{C} = \boldsymbol{B} \times \boldsymbol{A}$. Then from Problem 13, we will have

$$[\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{A})] \cdot \boldsymbol{B} = (\boldsymbol{A} \times \boldsymbol{C}) \cdot \boldsymbol{B} = \boldsymbol{C} \cdot (\boldsymbol{B} \times \boldsymbol{A}) = \boldsymbol{C} \cdot (-\boldsymbol{A} \times \boldsymbol{B}) = -\boldsymbol{C} \cdot (\boldsymbol{A} \times \boldsymbol{B}) = (\boldsymbol{A} \times \boldsymbol{B}) \cdot (\boldsymbol{A} \times \boldsymbol{B}).$$

Method 2: Use Problem 15 to write

$$[\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{A})] \cdot \boldsymbol{B} = [(\boldsymbol{A} \cdot \boldsymbol{A})\boldsymbol{B} - (\boldsymbol{A} \cdot \boldsymbol{B})\boldsymbol{A}] \cdot \boldsymbol{B} = (\boldsymbol{A} \cdot \boldsymbol{A})(\boldsymbol{B} \cdot \boldsymbol{B}) - (\boldsymbol{A} \cdot \boldsymbol{B})(\boldsymbol{A} \cdot \boldsymbol{B})$$

$$= |\boldsymbol{A}|^2 |\boldsymbol{B}|^2 - |\boldsymbol{A}|^2 |\boldsymbol{B}|^2 \cos^2\theta = |\boldsymbol{A}|^2 |\boldsymbol{B}|^2 \sin^2\theta = |\boldsymbol{A} \times \boldsymbol{B}|^2 = (\boldsymbol{A} \times \boldsymbol{B}) \cdot (\boldsymbol{A} \times \boldsymbol{B}).$$