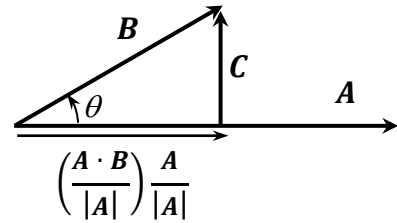


Problem 1-16) Consider the projection of \mathbf{B} onto the unit-vector $\mathbf{A}/|\mathbf{A}|$, that is,

$$\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}|} = \frac{|\mathbf{A}| |\mathbf{B}| \cos \theta}{|\mathbf{A}|} = |\mathbf{B}| \cos \theta.$$



Since the unit-vector $\mathbf{A}/|\mathbf{A}|$ is aligned with \mathbf{A} , the vector $\left(\frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}|}\right) \frac{\mathbf{A}}{|\mathbf{A}|}$ has length $|\mathbf{B}| \cos \theta$ and is parallel to \mathbf{A} ; in other words, it is the “shadow” of \mathbf{B} on \mathbf{A} . When this shadow projection is removed (i.e., subtracted) from \mathbf{B} , what remains is \mathbf{C} , which is perpendicular to \mathbf{A} . This can be shown directly, as follows:

$$\mathbf{A} \cdot \mathbf{C} = \mathbf{A} \cdot \left(\mathbf{B} - \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}|^2} \mathbf{A} \right) = (\mathbf{A} \cdot \mathbf{B}) - \frac{\mathbf{A} \cdot \mathbf{B}}{|\mathbf{A}|^2} (\mathbf{A} \cdot \mathbf{A}).$$

But $\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2$; therefore, $\mathbf{A} \cdot \mathbf{C} = \mathbf{A} \cdot \mathbf{B} - \mathbf{A} \cdot \mathbf{B} = 0$, which implies that the angle between \mathbf{A} and \mathbf{C} is 90° , that is, \mathbf{A} and \mathbf{C} are orthogonal to each other.
