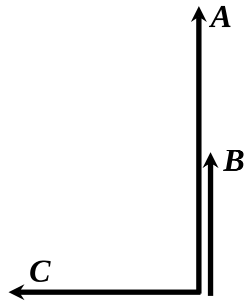
Problem 1-15) a) Consider the three vectors shown on the right-hand side. Since $\boldsymbol{A}$ and $\boldsymbol{B}$ are parallel, $\boldsymbol{A}\times\boldsymbol{B}=0$. Therefore, $(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C}=0$. In contrast, $\boldsymbol{B}\times\boldsymbol{C}$ has magnitude $BC=|\boldsymbol{B}||\boldsymbol{C}|$ and is perpendicular to the plane of the paper. Cross-multiplying $\boldsymbol{A}$ into $\boldsymbol{B}\times\boldsymbol{C}$ thus yields a vector of magnitude $ABC=|\boldsymbol{A}||\boldsymbol{B}||\boldsymbol{C}|$, which is anti-parallel with $\boldsymbol{C}$. Therefore, $(\boldsymbol{A}\times\boldsymbol{B})\times\boldsymbol{C}$ is not necessarily equal to $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})$.



b) $$\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})=(A_x\hat{\boldsymbol{x}}+A_y\hat{\boldsymbol{y}}+A_z\hat{\boldsymbol{z}})\times\left[(B_yC_z-B_zC_y)\hat{\boldsymbol{x}}+(B_zC_x-B_xC_z)\hat{\boldsymbol{y}}+(B_xC_y-B_yC_x)\hat{\boldsymbol{z}}\right]$$

$$\begin{aligned}&=[A_y(B_xC_y-B_yC_x)-A_z(B_zC_x-B_xC_z)]\hat{\boldsymbol{x}}\\&\quad+[A_z(B_yC_z-B_zC_y)-A_x(B_xC_y-B_yC_x)]\hat{\boldsymbol{y}}\\&\quad+[A_x(B_zC_x-B_xC_z)-A_y(B_yC_z-B_zC_y)]\hat{\boldsymbol{z}}\end{aligned}$$

$$\begin{aligned}&=(A_yC_y+A_zC_z)B_x\hat{\boldsymbol{x}}-(A_yB_y+A_zB_z)C_x\hat{\boldsymbol{x}}\\&\quad+(A_xC_x+A_zC_z)B_y\hat{\boldsymbol{y}}-(A_xB_x+A_zB_z)C_y\hat{\boldsymbol{y}}\\&\quad+(A_xC_x+A_yC_y)B_z\hat{\boldsymbol{z}}-(A_xB_x+A_yB_y)C_z\hat{\boldsymbol{z}}\end{aligned}$$

$$\begin{aligned}&=(A_xC_x+A_yC_y+A_zC_z)B_x\hat{\boldsymbol{x}}-(A_xB_x+A_yB_y+A_zB_z)C_x\hat{\boldsymbol{x}}\\&\quad+(A_xC_x+A_yC_y+A_zC_z)B_y\hat{\boldsymbol{y}}-(A_xB_x+A_yB_y+A_zB_z)C_y\hat{\boldsymbol{y}}\\&\quad+(A_xC_x+A_yC_y+A_zC_z)B_z\hat{\boldsymbol{z}}-(A_xB_x+A_yB_y+A_zB_z)C_z\hat{\boldsymbol{z}}\end{aligned}$$

$$=(\boldsymbol{A}\cdot\boldsymbol{C})(B_x\hat{\boldsymbol{x}}+B_y\hat{\boldsymbol{y}}+B_z\hat{\boldsymbol{z}})-(\boldsymbol{A}\cdot\boldsymbol{B})(C_x\hat{\boldsymbol{x}}+C_y\hat{\boldsymbol{y}}+C_z\hat{\boldsymbol{z}})$$

$$=(\boldsymbol{A}\cdot\boldsymbol{C})\boldsymbol{B}-(\boldsymbol{A}\cdot\boldsymbol{B})\boldsymbol{C}.$$

Geometric Interpretation: The vector $\boldsymbol{B}\times\boldsymbol{C}$ is perpendicular to the plane of $\boldsymbol{B}$ and $\boldsymbol{C}$. Therefore, when cross-multiplied into $\boldsymbol{A}$, the resulting vector must lie in the plane of $\boldsymbol{B}$ and $\boldsymbol{C}$. In other words, the vector $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})$ must be co-planar with $\boldsymbol{B}$ and $\boldsymbol{C}$. As such, it can be written as a linear combination of $\boldsymbol{B}$ and $\boldsymbol{C}$, that is, $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})=\alpha\boldsymbol{B}+\beta\boldsymbol{C}$. This is all that one can say about the $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C})$ triple cross-product. The fact that $\alpha=\boldsymbol{A}\cdot\boldsymbol{C}$ and $\beta=-\boldsymbol{A}\cdot\boldsymbol{B}$ comes out of the preceding calculations, with no obvious geometric interpretation.