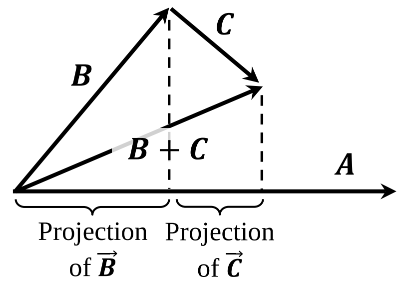
Problem 1-14)

a) $\boldsymbol{A}\cdot\boldsymbol{B} = |\boldsymbol{A}|\,|\boldsymbol{B}|\cos\theta = \boldsymbol{B}\cdot\boldsymbol{A}$.

b)
$$\begin{aligned}\boldsymbol{A}\cdot(\boldsymbol{B}+\boldsymbol{C}) &= \left(A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}\right)\cdot\left[(B_x+C_x)\hat{\boldsymbol{x}} + \left(B_y+C_y\right)\hat{\boldsymbol{y}} + (B_z+C_z)\hat{\boldsymbol{z}}\right]\\ &= A_x(B_x+C_x) + A_y\left(B_y+C_y\right) + A_z(B_z+C_z)\\ &= \left(A_xB_x + A_yB_y + A_zB_z\right) + \left(A_xC_x + A_yC_y + A_zC_z\right)\\ &= \boldsymbol{A}\cdot\boldsymbol{B} + \boldsymbol{A}\cdot\boldsymbol{C}.\end{aligned}$$

Geometric interpretation: The projection of $\boldsymbol{B}+\boldsymbol{C}$ on $\boldsymbol{A}$ has the same length as the projection of $\boldsymbol{B}$ on $\boldsymbol{A}$, plus that of $\boldsymbol{C}$ on $\boldsymbol{A}$. (Note: The vectors $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in the diagram are not necessarily in the same plane.)



c) $\boldsymbol{A}\times\boldsymbol{B}$ and $\boldsymbol{B}\times\boldsymbol{A}$ are both perpendicular to the plane defined by $\boldsymbol{A}$ and $\boldsymbol{B}$, and both have a magnitude equal to the area of the parallelogram formed by $\boldsymbol{A}$ and $\boldsymbol{B}$. The right-hand rule, however, gives opposite directions to $\boldsymbol{A}\times\boldsymbol{B}$ and $\boldsymbol{B}\times\boldsymbol{A}$. Therefore, $\boldsymbol{A}\times\boldsymbol{B} = -\boldsymbol{B}\times\boldsymbol{A}$.

d)
$$\begin{aligned}\boldsymbol{A}\times(\boldsymbol{B}+\boldsymbol{C}) &= \left(A_x\hat{\boldsymbol{x}} + A_y\hat{\boldsymbol{y}} + A_z\hat{\boldsymbol{z}}\right)\times\left[(B_x+C_x)\hat{\boldsymbol{x}} + \left(B_y+C_y\right)\hat{\boldsymbol{y}} + (B_z+C_z)\hat{\boldsymbol{z}}\right]\\ &= \left[A_y(B_z+C_z) - A_z\left(B_y+C_y\right)\right]\hat{\boldsymbol{x}} + \left[A_z(B_x+C_x) - A_x(B_z+C_z)\right]\hat{\boldsymbol{y}}\\ &\quad + \left[A_x\left(B_y+C_y\right) - A_y(B_x+C_x)\right]\hat{\boldsymbol{z}}\\ &= \left(A_yB_z - A_zB_y\right)\hat{\boldsymbol{x}} + \left(A_yC_z - A_zC_y\right)\hat{\boldsymbol{x}}\\ &\quad + (A_zB_x - A_xB_z)\hat{\boldsymbol{y}} + (A_zC_x - A_xC_z)\hat{\boldsymbol{y}}\\ &\quad + \left(A_xB_y - A_yB_x\right)\hat{\boldsymbol{z}} + \left(A_xC_y - A_yC_x\right)\hat{\boldsymbol{z}}\\ &= \boldsymbol{A}\times\boldsymbol{B} + \boldsymbol{A}\times\boldsymbol{C}.\end{aligned}$$

e) $(\boldsymbol{A}+\boldsymbol{B})\times\boldsymbol{C} = -\boldsymbol{C}\times(\boldsymbol{A}+\boldsymbol{B}) = -\boldsymbol{C}\times\boldsymbol{A} - \boldsymbol{C}\times\boldsymbol{B} = \boldsymbol{A}\times\boldsymbol{C} + \boldsymbol{B}\times\boldsymbol{C}$.