

**Opti 501****Solutions**

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**Problem 13)** The cross-product  $\mathbf{A} \times \mathbf{B}$  has a magnitude equal to the area of the base of the parallel-piped, and is perpendicular to this base. Dot-multiplication with  $\mathbf{C}$  then yields the product of the base area with the projection of  $\mathbf{C}$  on the normal to the base (i.e., the height of the parallel-piped). The product of the height and the base area is thus equal to the volume.

$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$  because all three combinations represent the volume of the *same* parallel-piped. Note that the volume calculated in this way may have a minus sign and so the order of cross-multiplication is important.

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