Problem 1-10)

a)
$$f(x) = \sqrt{1+x} = (1+x)^{1/2},$$
$$f'(x) = ½(1+x)^{-1/2},$$
$$f''(x) = -¼(1+x)^{-3/2},$$
$$f'''(x) = ⅜(1+x)^{-5/2}.$$

The Taylor series expansion of $\sqrt{1+x}$ around $x = 0$ is thus given by

$$\sqrt{1+x} = 1 + ½\left(\frac{x}{1!}\right) - ¼\left(\frac{x^2}{2!}\right) + ⅜\left(\frac{x^3}{3!}\right) - \cdots = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots.$$

We may now write

$x = 0.1$: $\sqrt{1+x} = 1.0488,$ $1 + ½x = 1.05.$

$x = 0.2$: $\sqrt{1+x} = 1.0954,$ $1 + ½x = 1.10.$

$x = 0.3$: $\sqrt{1+x} = 1.1402,$ $1 + ½x = 1.15.$

$x = 0.4$: $\sqrt{1+x} = 1.1832,$ $1 + ½x = 1.20.$

$x = 0.5$: $\sqrt{1+x} = 1.2247,$ $1 + ½x = 1.25.$

b) Writing $(1+x) = A\exp(\mathrm{i}\varphi)$, we will have $\sqrt{1+x} = \sqrt{A}\exp(½\mathrm{i}\varphi)$. Since, to first order in $|x|$, we have $\sqrt{1+x} \cong 1 + ½x$, it follows that, for small $|x|$,

$$|1 + ½x| \cong \sqrt{A},$$
$$\text{Phase-angle of } (1 + ½x) \cong \varphi/2.$$

The diagram below is instructive.