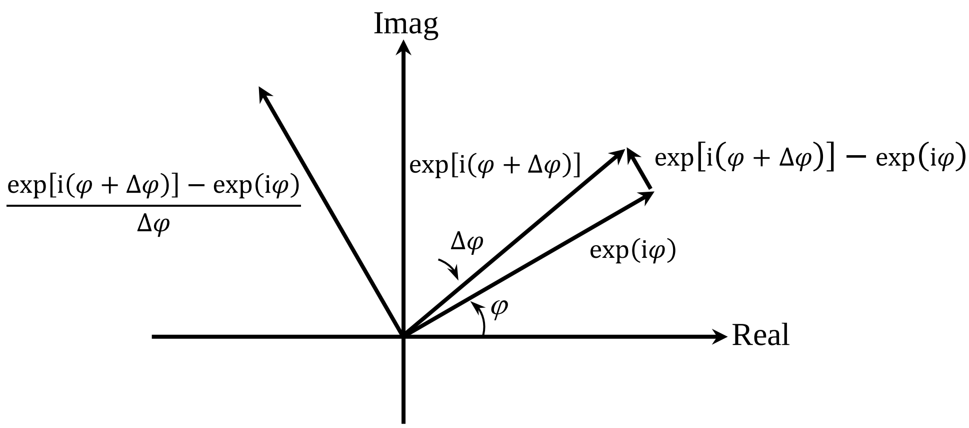
Problem 1-9) Each of the complex numbers $\exp(\mathrm{i}\varphi)$ and $\exp[\mathrm{i}(\varphi+\Delta\varphi)]$ has unit length. For small $\Delta\varphi$, the length of $\{\exp[\mathrm{i}(\varphi+\Delta\varphi)]-\exp(\mathrm{i}\varphi)\}$ is equal to $\Delta\varphi$ (see the figure below). Also, the vector representing $\{\exp[\mathrm{i}(\varphi+\Delta\varphi)]-\exp(\mathrm{i}\varphi)\}$ is orthogonal to $\exp(\mathrm{i}\varphi)$, because it is tangent to a circle of radius 1 centered at the origin. Normalizing $\{\exp[\mathrm{i}(\varphi+\Delta\varphi)]-\exp(\mathrm{i}\varphi)\}$ by $\Delta\varphi$ thus yields a complex number of unit length which makes an angle of $\varphi+90°$ with the *x*-axis.



Analytically, we may write

$$\frac{d}{d\varphi}\exp(\mathrm{i}\varphi)=\mathrm{i}\exp(\mathrm{i}\varphi)=\exp(\mathrm{i}\pi/2)\exp(\mathrm{i}\varphi)=\exp\left[\mathrm{i}\left(\varphi+\frac{\pi}{2}\right)\right],$$

which is also a complex number of unit length that makes an angle of $\varphi+90°$ with the *x*-axis.