

Opti 501

Solutions

1-8) a) $A = |A| \exp(i\phi) = |A| \cos \phi + i|A| \sin \phi$. Therefore,

$$\begin{aligned} A \exp(i\theta) &= |A| (\cos \phi + i \sin \phi)(\cos \theta + i \sin \theta) \\ &= |A| [(\cos \phi \cos \theta - \sin \phi \sin \theta) + i(\cos \phi \sin \theta + \sin \phi \cos \theta)] \\ &= |A| [\cos(\phi + \theta) + i \sin(\phi + \theta)] = |A| \exp[i(\phi + \theta)]. \end{aligned}$$

b)
$$\begin{aligned} \frac{A}{\exp(i\theta)} &= \frac{|A| (\cos \phi + i \sin \phi)}{\cos \theta + i \sin \theta} = \frac{|A| (\cos \phi + i \sin \phi)(\cos \theta - i \sin \theta)}{(\cos \theta + i \sin \theta)(\cos \theta - i \sin \theta)} \\ &= \frac{|A| [(\cos \phi \cos \theta + \sin \phi \sin \theta) + i(\sin \phi \cos \theta - \cos \phi \sin \theta)]}{\cos^2 \theta + \sin^2 \theta} \\ &= |A| [\cos(\phi - \theta) + i \sin(\phi - \theta)] = |A| \exp[i(\phi - \theta)]. \end{aligned}$$