Opti 501 **Solutions**

1-8) a) $A = |A| \exp(\mathrm{i}\phi) = |A| \cos\phi + \mathrm{i}|A| \sin\phi$. Therefore,

$$A\exp(\mathrm{i}\theta) = |A|(\cos\phi + \mathrm{i}\sin\phi)(\cos\theta + \mathrm{i}\sin\theta).$$

$$= |A|[(\cos\phi\cos\theta - \sin\phi\sin\theta) + \mathrm{i}(\cos\phi\sin\theta + \sin\phi\cos\theta)]$$

$$= |A|[\cos(\phi+\theta) + \mathrm{i}\sin(\phi+\theta)] = |A|\exp[\mathrm{i}(\phi+\theta)].$$

b)
$$\frac{A}{\exp(\mathrm{i}\theta)} = \frac{|A|(\cos\phi + \mathrm{i}\sin\phi)}{\cos\theta + \mathrm{i}\sin\theta} = \frac{|A|(\cos\phi + \mathrm{i}\sin\phi)(\cos\theta - \mathrm{i}\sin\theta)}{(\cos\theta + \mathrm{i}\sin\theta)(\cos\theta - \mathrm{i}\sin\theta)}$$

$$= \frac{|A|[(\cos\phi\cos\theta + \sin\phi\sin\theta) + \mathrm{i}(\sin\phi\cos\theta - \cos\phi\sin\theta)]}{\cos^2\theta + \sin^2\theta}$$

$$= |A|[\cos(\phi-\theta) + \mathrm{i}\sin(\phi-\theta)] = |A|\exp[\mathrm{i}(\phi-\theta)].$$
