Problem 1-6)

$$f(x) = 1 + x + x^2 + x^3 + \cdots.$$

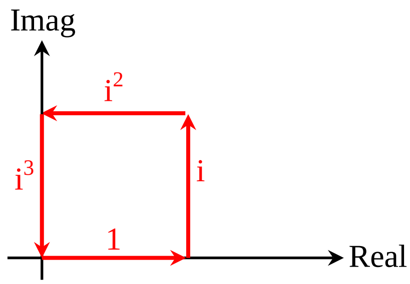
Multiply and divide by $(1 - x)$ to obtain:

$$f(x) = \frac{(1-x)(1+x+x^2+x^3+\cdots)}{1-x} = \frac{(1-x) + (x-x^2) + (x^2-x^3) + \cdots + (x^n-x^{n+1}) + \cdots}{1-x}$$

$$= \frac{1 - x^{n+1}}{1-x} \qquad \leftarrow \text{ To be evaluated in the limit when } n \to \infty.$$

Let $x = x_1 + \mathrm{i}x_2 = |x| \exp(\mathrm{i}\varphi)$. If $|x| < 1$, we will have $|x^{n+1}| = |x|^{n+1} \to 0$ when $n \to \infty$, irrespective of the value of φ. Therefore, $f(x) = 1/(1 - x)$ provided that $|x| < 1$.

In the figure below, the series $1 + \mathrm{i} + \mathrm{i}^2 + \mathrm{i}^3 + \cdots$ is seen to keep going around the square without ever converging to a fixed point. This is because $|\mathrm{i}| = 1$, which is *not* strictly less than 1.



In contrast, the series $1 + (0.99\mathrm{i}) + (0.99\mathrm{i})^2 + (0.99\mathrm{i})^3 + \cdots$ depicted below, converges to the center of the square, close to the point $(½ + ½\mathrm{i})$, as may be seen from the following closed form:

$$f(x) = \frac{1}{1-x} = \frac{1}{1 - 0.99\mathrm{i}} = \frac{1 + 0.99\mathrm{i}}{1 + 0.99^2} \cong ½ + ½\mathrm{i}.$$