Problem 1-5) a) $\frac{d}{dx}\ln x = x^{-1}$; $\frac{d^2}{dx^2}\ln x = -x^{-2}$; $\frac{d^3}{dx^3}\ln x = 2x^{-3}$; $\frac{d^4}{dx^4}\ln x = -3!\,x^{-4}$.

Taylor series expansion around $x = x_0$: $\ln x = \ln x_0 + (\ln x_0)'\frac{(x-x_0)}{1!} + (\ln x_0)''\frac{(x-x_0)^2}{2!} + \cdots$.

Now, setting $x_0 = 1$ yields $\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$.

b) Setting $x = ½$, we find $\ln(½) = -½ - \frac{(½)^2}{2} - \frac{(½)^3}{3} - \frac{(½)^4}{4} - \cdots$.

Since $\ln(½) = -\ln 2$, we will have $\ln 2 = \frac{1}{2} + \frac{1}{2\times 2^2} + \frac{1}{3\times 2^3} + \frac{1}{4\times 2^4} + \cdots$.

Similarly, setting $x = 2$, we find $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots$.

These are two different expansions for ln 2.

c) Setting $x = 1 + \mathrm{i} = \sqrt{2}\exp(\mathrm{i}\pi/4)$, we find $\ln(1+\mathrm{i}) = (\ln\sqrt{2}) + \mathrm{i}(\pi/4)$. Therefore,

$$\ln(1+\mathrm{i}) = \frac{1}{2}\ln 2 + \mathrm{i}\frac{\pi}{4}.$$

Taylor series expansion: $\ln(1+\mathrm{i}) = \mathrm{i} - \frac{\mathrm{i}^2}{2} + \frac{\mathrm{i}^3}{3} - \frac{\mathrm{i}^4}{4} + \cdots = \mathrm{i} + \frac{1}{2} - \frac{\mathrm{i}}{3} - \frac{1}{4} + \cdots$.

Therefore,

$$\frac{1}{2}\ln 2 + \mathrm{i}\frac{\pi}{4} = \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \cdots\right) + \mathrm{i}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right) + \mathrm{i}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$$

see part (b) → $$= \frac{1}{2}\ln 2 + \mathrm{i}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right).$$

We thus find $\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots\right)$.