Problem 1-4) The Taylor series expansion of $f(x)$ around $x = x_0$ is given by

$$f(x) = f(x_0) + f'(x_0)\frac{(x-x_0)}{1!} + f''(x_0)\frac{(x-x_0)^2}{2!} + \cdots + f^{(n)}(x_0)\frac{(x-x_0)^n}{n!} + \cdots .$$

Considering that $d(\sin x)/dx = \cos x$ and $d(\cos x)/dx = -\sin x$, we may write

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots ,$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots .$$

As for the complex exponential function $\exp(\mathrm{i}x)$, we have

$$\frac{d}{dx}\exp(\mathrm{i}x) = \mathrm{i}\exp(\mathrm{i}x),$$

$$\frac{d^2}{dx^2}\exp(\mathrm{i}x) = \mathrm{i}^2\exp(\mathrm{i}x) = -\exp(\mathrm{i}x),$$

$$\frac{d^3}{dx^3}\exp(\mathrm{i}x) = -\mathrm{i}\exp(\mathrm{i}x).$$

Therefore,

$$\begin{aligned}\exp(\mathrm{i}x) &= 1 + \mathrm{i}x - \frac{x^2}{2!} - \mathrm{i}\frac{x^3}{3!} + \frac{x^4}{4!} + \mathrm{i}\frac{x^5}{5!} - \frac{x^6}{6!} + \cdots \\ &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right) + \mathrm{i}\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right) \\ &= \cos x + \mathrm{i}\sin x.\end{aligned}$$