Problem 1-3)

a) $$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots.$$

Set $x = 1$. You will find $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{n!} + \cdots$.

b) $$e^x = e^{x_1} + e^{x_1}\frac{(x-x_1)}{1!} + e^{x_1}\frac{(x-x_1)^2}{2!} + e^{x_1}\frac{(x-x_1)^3}{3!} + \cdots + e^{x_1}\frac{(x-x_1)^n}{n!} + \cdots.$$

Note that the n^{th} derivative of e^x is also e^x for all values of n (i.e., $n = 1, 2, 3, ...$). That is why all the coefficients in the above expansion are e^{x_1}, which is the value of the n^{th} derivative of e^x at $x = x_1$. Setting $x = x_1 + x_2$, we find

$$e^{x_1+x_2} = e^{x_1}\left(1 + \frac{x_2}{1!} + \frac{x_2^2}{2!} + \frac{x_2^3}{3!} + \cdots + \frac{x_2^n}{n!} + \cdots\right) = e^{x_1}e^{x_2}.$$